Conditional Risk

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Conditional Risk: The CAPM is About More than Average Betas

The CAPM is about compensation for market risk

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We often calculate α wrt. unconditional beta

$$\alpha = E[r^{excess}] - \beta\lambda$$

Conditional Risk: The CAPM is About More than Average Betas

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We often calculate α wrt. unconditional beta

$$\alpha = E[r^{excess}] - \beta\lambda$$

But riskiness also depends on time variation in betas

- If β is higher when λ is high or variance is low, risk is higher
- CAPM predicts higher required returns from such **conditional risk**

Conventional Wisdom on Conditional Risk

Lewellen and Nagel (2006) often used as motivation for conventional wisdom

1. Market betas do not vary by enough to ***possibly*** matter for required returns and alphas
2. No need to worry about conditional risk

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1. Market betas ***do*** vary by enough to matter for required returns and alphas
   - Beta, risk premia, and variance are more volatile than previously thought
   - We disagree with past arguments
2. Conditional risk matters even if it cannot explain everything
   - Just like the CAPM matters even though the model fails

Road-map and Overview of Results

1 Revisiting the scope for conditional risk

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1. Revisiting the scope for conditional risk
   - Conditional risk can, in theory, explain 5 pp. of return on risk factors

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   - Conditional risk can, in theory, explain 5 pp. of return on risk factors

2. New methods to account for conditional risk
   - New conditional risk factors
   - Easy way to quantify impact of conditional risk

Road-map and Overview of Results

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   - Conditional risk can, in theory, explain 5 pp. of return on risk factors
2. New methods to account for conditional risk
   - New conditional risk factors
   - Easy way to quantify impact of conditional risk
3. Empirical analysis of conditional risk in equity factors

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   - Conditional risk can, in theory, explain 5 pp. of return on risk factors

2. New methods to account for conditional risk
   - New conditional risk factors
   - Easy way to quantify impact of conditional risk

3. Empirical analysis of conditional risk in equity factors
   - Most major risk factors load on conditional risk
   - Conditional risk explains $\sim$ 2 pp. alpha on value and momentum factors (in recent sample)
   - Pervasive global role for conditional risk

1. Revisiting the Theoretical Potential for Conditional Risk

Recap: Conditional Risk in Theory

According to the CAPM,

$$E[r] - r^f = \underbrace{\beta\lambda}_{\text{unconditional risk}} + \underbrace{\text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)}$$

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- $E[r] - r^f$ is the unconditional excess return on a given asset
- β and λ are the unconditional beta and market risk premium

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- β and λ are the unconditional beta and market risk premium

Conditional risk

- Influences required returns when β_t varies over time
- Strength depends on volatility of β_t, λ_t, σ_t^{mkt}

How Much Can Conditional Risk Explain?

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

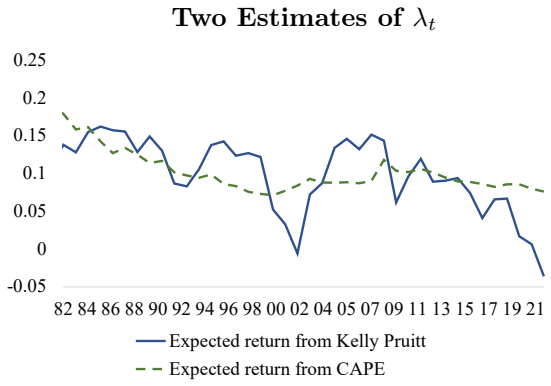
Lewellen and Nagel (2006) argues

1. β and λ_t do not vary very much
2. And movement in σ_t^{mkt} cancels out with λ_t

But recent papers suggest

- That β_t and λ_t are highly volatile (Kelly, Moskowitz, and Pruitt, 2021; Kelly and Pruitt, 2013)
- Movement in σ_t^{mkt} does not cancel out with λ_t (Moreira and Muir, 2017)

λ_t is More Volatile Than Previously Thought



- λ_t more volatile than previously thought
- Particularly at high frequency

Quantifying the Role of Conditional Risk: Market Timing

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

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$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

$\rho_{\beta_t, E_t} = 1$		σ_{E_t} 0.2	0.4	0.6
	0.2	0.24	0.48	0.72
σ_{β_t}	0.3	0.72	1.44	2.16
	0.4	0.96	1.92	2.88

Takeaways

- Market timing alone can explain 2.88% return
- Lewellen and Nagel consider this an upper bound on conditional risk, discarding potential effects of volatility

Quantifying the Role of Conditional Risk: Volatility Timing

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

Quantifying the Role of Conditional Risk: Volatility Timing

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

$\rho_{\beta_t, var_t} = -1$		σ_{var_t} 0.2	0.4	0.6
	0.2	0.60	1.20	1.80
σ_{β_t}	0.3	1.80	3.60	5.40
	0.4	2.40	4.80	7.20

Takeaways

- Volatility timing alone can explain $> 7\%$ return
- If risk premia were constant, this would be the true upper bound on conditional risk

Quantifying the Role of Conditional Risk: Combined Effect

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

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$\sigma_{E_t} = 0.5$		σ_{var_t} 0.2	0.4	0.6
	0.2	1.20	1.80	2.40
σ_{β_t}	0.3	1.80	2.70	3.60
	0.4	2.40	3.60	4.80

Takeaways

- When both risk premia and volatility vary over time, there is likely some canceling out
- But risk premia and variance are close to uncorrelated, so little canceling out
- If uncorrelated, volatility and market timing can explain 5% of return to risk factors

2. Conditional-Risk Factors

Conditional Risk Factors

We define a "conditional-risk factor"

$$c_{t+1} = \tilde{r}_{t+1}(b_t - b)$$

- Where $\tilde{r}_{t+1}$ is the realized excess return on the market
- b_t and b are the conditional and unconditional prices of risk $\left(\frac{\lambda_t}{\text{var}_t(r^{\text{mkt}})}\right)$

  $\Rightarrow$ it is a market timing strategy based on conditional price of risk

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Proposition 1 (conditional risk in factor models)
The unconditional expected excess return on an asset i is given by

$$E[r^i_{t+1}] = \tilde{\beta}\lambda + \beta_c\lambda_c$$

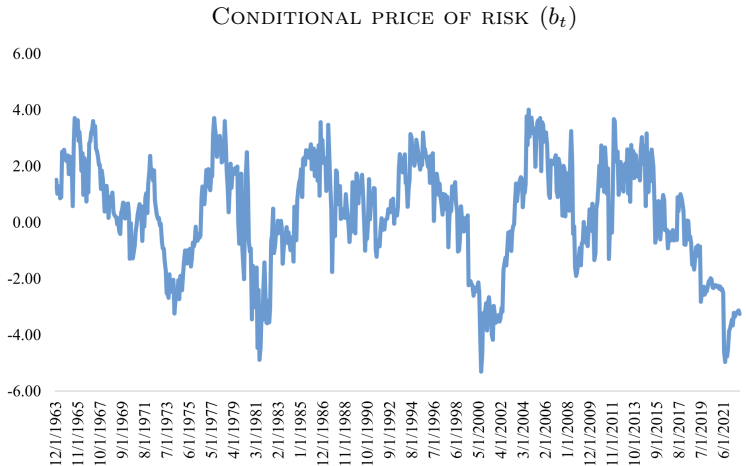
where β_c is the beta wrt. the conditional risk factor and λ_c is the risk premium

Conditional-Risk Factors: Empirical Implementation

We need to measure λ_t and $\text{var}_t(r^{\text{mkt}})$ to construct b_t

- For λ_t we use Kelly and Pruitt (2013)
  - R^2 of 1.3% in monthly regressions
  - Generalize to global sample and find median R^2 of 1.5%
- For $\text{var}_t(r^{\text{mkt}})$, we use an AR(1) for realized returns
  - R^2 20%-50%

Conditional-Risk Factors: Empirical Implementation



Why Should We Care About Conditional Risk?

Two Reasons We Care About Conditional Risk

1. Conditional risk is *real* risk
   - Conditional risk reflects that the having a high beta during bad time is riskier than having high betas during good times
     (With good times defined as high expected returns relative to variance)

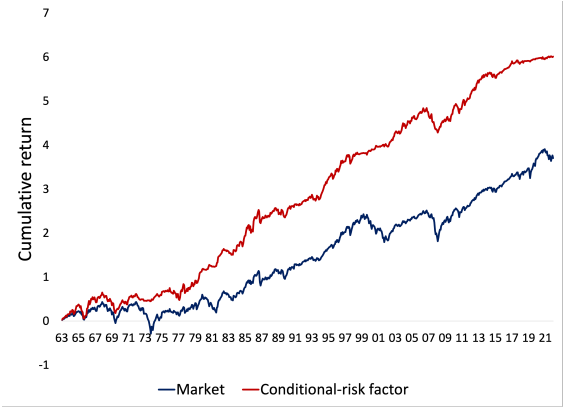
Two Reasons We Care About Conditional Risk

1. Conditional risk is *real* risk
   - Conditional risk reflects that the having a high beta during bad time is riskier than having high betas during good times
     (With good times defined as high expected returns relative to variance)

2. Conditional risk reflects a market timing strategy that can easily be replicated
   - The unconditional alpha associated with conditional risk can be valuable to some investors
   - But you can get that easily by investing directly in the conditional risk factor

Conditional Risk as a Market Timing Strategy

Cumulative returns to market and conditional-risk factor



- Conditional-risk factor has an annual alpha of 5.31% (t-stat 4.31)
- Sharpe ratio of 0.64 (vs. market of 0.4)

3. Empirical Analysis of Conditional Risk in Equity Factors

Conditional Risk in Equity Factors

	HML	RMW	CMA	UMD	BAB	TAN
Panel A: Full U.S. Sample						
Alpha in unconditional CAPM						
Alpha in conditional model						
Compensation for conditional risk						
Fraction of alpha explained by conditional risk						
Compensation for unconditional risk						
Observations						
Adjusted R^2						

Conditional Risk in Equity Factors

	HML	RMW	CMA	UMD	BAB	TAN
Panel A: Full U.S. Sample						
Alpha in unconditional CAPM	**4.58**					
	(2.67)					
Alpha in conditional model	**3.33**					
	(2.08)					
Compensation for conditional risk	**1.25**					
	(2.12)					
Fraction of alpha explained by conditional risk	0.27					
Compensation for unconditional risk	-0.86					
Observations	708					
Adjusted R^2	0.11					

Conditional Risk in Equity Factors

	HML	RMW	CMA	UMD	BAB	TAN
Panel A: Full U.S. Sample						
Alpha in unconditional CAPM	**4.58**	**3.95**	**4.70**	**8.63**	**9.99**	**4.97**
	(2.67)	(3.43)	(4.54)	(4.93)	(5.11)	(7.91)
Alpha in conditional model	**3.33**	**3.57**	**3.80**	**8.05**	**8.95**	**4.24**
	(2.08)	(3.55)	(4.02)	(4.61)	(4.91)	(7.87)
Compensation for conditional risk	**1.25**	0.38	**0.90**	0.58	1.04	**0.73**
	(2.12)	(0.85)	(2.68)	(0.67)	(1.47)	(1.98)
Fraction of alpha explained by conditional risk	0.27	0.10	0.19	0.07	0.10	0.15
Compensation for unconditional risk	-0.86	-0.58	-1.04	-0.99	-0.33	0.58
Observations	708	708	708	708	708	708
Adjusted R^2	0.11	0.15	0.22	0.29	0.03	0.30

Conditional Risk in Equity Factors: Late Sample

Focus on post-1996:

- Risk premia known to be highly volatile post 1996
- Roughly out-of-sample relative to Lewellen and Nagel (2006)

Conditional Risk in Equity Factors: Late Sample

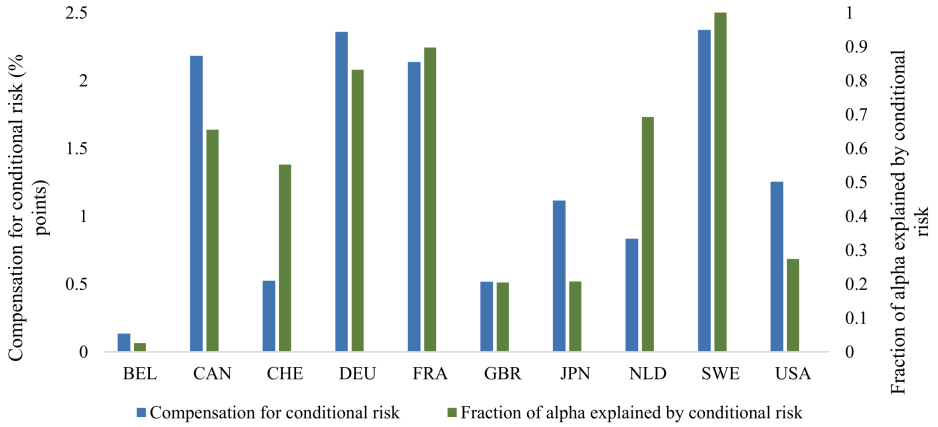
	HML	RMW	CMA	UMD	BAB	TAN
Panel B: Post 1996 U.S. Sample						
Alpha in unconditional CAPM	2.45					
	(0.77)					
Alpha in conditional model	0.43					
	(0.15)					
Compensation for conditional risk	2.02					
	(1.63)					
Fraction of alpha explained by conditional risk	0.83					
Compensation for unconditional risk	-0.48					
Observations	324					
Adjusted R^2	0.15					

Conditional Risk in Equity Factors: Late Sample

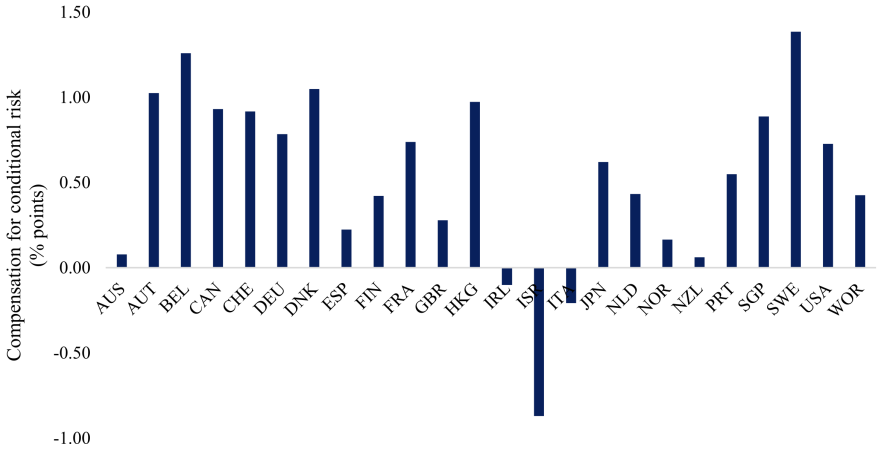
	HML	RMW	CMA	UMD	BAB	TAN
Panel B: Post 1996 U.S. Sample						
Alpha in unconditional CAPM	2.45	**6.29**	**4.43**	**7.25**	**10.42**	**5.23**
	(0.77)	(3.30)	(2.50)	(2.62)	(2.92)	(4.44)
Alpha in conditional model	0.43	**5.62**	2.61	5.19	**9.19**	**3.76**
	(0.15)	(3.33)	(1.64)	(1.75)	(2.88)	(3.81)
Compensation for conditional risk	2.02	0.68	**1.82**	2.05	1.24	**1.47**
	(1.63)	(0.77)	(2.42)	(1.19)	(1.38)	(2.25)
Fraction of alpha explained by conditional risk	0.83	0.11	0.41	0.28	0.12	0.28
Compensation for unconditional risk	-0.48	-1.64	-1.17	-2.63	-1.74	0.17
Observations	324	324	324	324	324	324
Adjusted R^2	0.15	0.22	0.26	0.32	0.10	0.25

- Conditional risk in fact explains value alpha in the data since Lewellen and Nagel (2006)
- Conditional risk also matters for other risk factors
- Conditional risk matters more than unconditional market risk

Conditional Risk in Equity Factors: Global Evidence For HML



Conditional Risk in Equity Factors: Global Evidence for Tangency Portfolio



- Tangency portfolio loads in conditional risk in (almost) all developed countries
- Conditional risk explains $\sim$ 1 percentage point of return

Understanding the Drivers of Conditional Risk

Recall that conditional risk depends on two terms

$$\text{Conditional risk} = \text{cov}\left(\beta_t; \lambda_t - b \times \text{var}_t(r^{\text{mkt}})\right)$$

- First part is about market timing
- Second part is about volatility timing

Understanding the Drivers of Conditional Risk

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Two new factors allow us to quantify the role of the two terms

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Two new factors allow us to quantify the role of the two terms

$$c^e_{t+1} = r_{t+1}\left(b_t - \frac{\lambda}{\text{var}_t(r^{\text{mkt}}_{t+1})}\right)$$
$$c^v_{t+1} = -r_{t+1}\left(b - \frac{\lambda}{\text{var}_t(r^{\text{mkt}}_{t+1})}\right).$$

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Proposition 3 shows that

$$\text{cov}(r^i_{t+1}; c^e_{t+1}) = \text{cov}\left(\beta_t; \lambda_t\right)$$
$$\text{cov}(r^i_{t+1}; c^v_{t+1}) = \text{cov}\left(\beta_t; -b\,\text{var}_t(r^{\text{mkt}}_{t+1})\right).$$

Understanding the Drivers of Conditional Risk

	HML	RMW	CMA	UMD	BAB	TAN
Panel A: U.S. sample						
Loading on conditional expected return factor	0.30	0.06	**0.26**	0.40	**0.53**	0.21
	(1.92)	(0.67)	(2.77)	(1.74)	(2.43)	(1.50)
Loading on conditional market variance factor	−0.18	0.24	0.02	**0.94**	**0.55**	**0.27**
	(−0.78)	(1.86)	(0.23)	(3.10)	(2.13)	(2.29)
Panel B: Global sample						
Loading on conditional expected return factor	**0.64**	0.00	**0.46**	0.06	**0.57**	0.15
	(2.65)	(0.57)	(3.21)	(0.79)	(3.04)	(0.86)
Loading on conditional market variance factor	−0.06	0.01	**0.15**	**0.87**	0.18	0.14
	(−0.12)	(1.08)	(2.42)	(2.67)	(0.99)	(1.04)

- Recall: Lewellen and Nagel (2006) argue that market and volatility timing should cancel out
- Empirically we find the opposite: almost all strategies load both on terms
- Helps explain why conditional risk matters so much

Conditional Risk in Multifactor Models

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Conditional risk in multifactor models

- Trading strategies like HML often used as risk factors on right-hand side (Fama and French, 1993)
- Idea is to test whether a given asset can be "explained" by HML exposure
- In these exercises, we often consider unconditional betas wrt. HML
- But beta of test asset wrt. HML may vary over time $\Rightarrow$ conditional risk!

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- In these exercises, we often consider unconditional betas wrt. HML
- But beta of test asset wrt. HML may vary over time $\Rightarrow$ conditional risk!

Implementation

- We design conditional-risk factors for each of the major trading strategies
- Test how much of the alpha to the tangency portfolio that these conditional risk factors explain

Conditional Risk in Multifactor Models

	Tangency portfolio				
Factor used on the right-hand side:	HML	RMW	CMA	UMD	BAB
Alpha without controls for conditional factor risk	**4.26**	**4.03**	**3.26**	**3.72**	**2.33**
	(8.47)	(8.23)	(7.33)	(7.88)	(6.11)
Alpha with controls for conditional factor risk	**4.06**	**4.07**	**3.14**	**3.69**	**2.47**
	(8.44)	(8.81)	(7.62)	(7.73)	(6.46)
Compensation for conditional factor k risk	0.20	−0.04	0.12	0.02	−0.14
Fraction of alpha explained by conditional risk conditional risk:	0.05	−0.01	0.04	0.01	−0.06
Observations	708	708	708	708	708
Adjusted R2	0.31	0.36	0.50	0.35	0.61

- Conditional risk with respect to other factors than the market is not important
- The market appears unique in terms of importance of conditional risk

Conclusion

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2. New methods to account for conditional risk
   - New conditional risk factors
   - Easy way to quantify impact of conditional risk

3. Conditional risk matters for the major equity risk factors
   - Most major risk factors load on conditional risk
   - Conditional risk explains $\sim$ 2 pp. alpha on value and momentum factors (in recent sample)
   - Pervasive global role for conditional risk