

Interest Rates and Equity Valuations*

Niels Joachim Gormsen and Eben Lazarus

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Abstract

How do changes in interest rates affect equity valuations? The impact depends on the underlying driver of the rate change. We decompose real-rate changes into three drivers: expected growth, risk, and “pure discounting.” Only pure-discounting shocks should transmit one-for-one to valuations, with weak-to-negative transmission from growth and risk. Using survey forecasts and asset prices, we estimate that pure-discounting shocks indeed transmit one-for-one to equity and explain 80% of cross-country valuation changes since 1990. Only 35% of the post-1990 U.S. rate decline, however, reflects pure discounting. We use our decomposition to study cross-sectional rate exposures, the equity premium, and monetary policy.

KEYWORDS: Stock prices, interest rates, duration, long-term growth

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1. Introduction

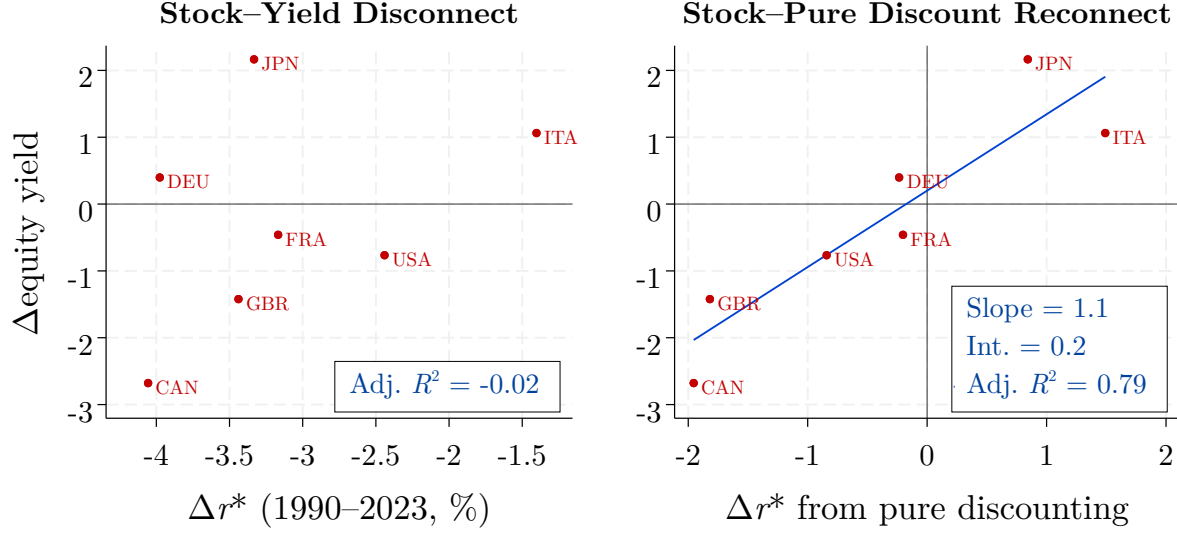
Advanced economies’ long-term interest rates have declined significantly in recent decades. How do such changes in rates transmit to equity valuations? A potentially tempting line of reasoning is to assume that equity discount rates move one-for-one with interest rates; since equity is a long-term asset whose value is highly sensitive to discount rates, this assumption implies that stock valuations should have increased substantially as a result of the secular decline in interest rates. High-level evidence might appear consistent with this view: in the U.S., for example, the market’s equity yield has declined substantially in recent decades, corresponding to a large increase in equity valuations over this period.

Empirically, however, there is no clear relationship between long-term changes in equity valuations and interest rates. The left panel of [Figure 1](#) presents one view of this stock–yield disconnect: across G7 economies, the change in a country’s equity yield since 1990 is effectively completely unrelated to that country’s change in the trend long-term real rate r^* , described further below. In addition to the example in [Figure 1](#), it is well known that the correlation between stock and bond returns is weak and often negative (e.g., [Campbell, Sunderam, and Viceira 2017](#)) — another example of the stock–yield disconnect.

The apparent stock–yield disconnect arises because the interest-rate sensitivity of stock prices is more complicated than alluded to in our opening paragraph. Interest rates are determined endogenously and may decline for multiple possible structural reasons, each of which may affect equity differently. Interest rates may, for instance, decline because of a decrease in expected growth rates in the economy, which — keeping all else constant — will decrease equity prices and thus mute the effects of the decline in interest rates on equity.

In this paper, we provide a framework and measurement approach to control for the underlying drivers of interest-rate movements and estimate the interest-rate sensitivity of equity prices. We start with a simple but general theoretical decomposition under which any change in trend real rates can be split into three mutually exclusive shocks: a change in expected growth, a change in uncertainty, and a “pure discounting” shock akin to a change in the rate of time preference. We then characterize how these shocks transmit to equity yields, the inverse of equity valuation ratios. The pure discounting shock transmits one-for-one from rates to equity yields, inducing perfect comovement between stocks and duration-matched bonds. But the remaining terms induce a weak and ambiguous relationship between stocks and bonds: a growth-rate shock affects both equity discount rates and cash-flow growth, while an uncertainty shock causes interest rates and equity risk premia to move in opposing directions. Isolating the pure discounting component of real rates is therefore key for understanding how much any given change in interest rates has passed through to equities.

Figure 1: Preview of Main Results: Long-Term Decomposition



Notes: This figure plots the country-level changes in equity yields against changes in trend interest rates (left panel) and against estimated changes in the pure discounting component of interest rates (right panel), for G7 economies. The sample is 1990–2023, or the longest available span for the given country. Measurement details for all variables are provided in [Section 3](#).

We next implement the decomposition empirically using a combination of survey data and option prices to estimate the underlying structural drivers. The punchline of our empirical implementation is that most of the secular changes in stock valuations over the past 35 years can be explained by movements in the pure discounting part of interest rates. This result is illustrated in the right panel of [Figure 1](#), which shows that 80% of the changes in valuation ratios of equities in G7 countries can be explained by changes in the pure discounting part of interest rates. This pure discounting component is estimated purely from our decomposition for interest rates, so there is nothing mechanical about the tight fit in explaining almost the entirety of the country-by-country change in equity valuations over recent decades.

The pure discounting part of interest rates also explains a sizable share of fluctuations in stock prices at higher frequencies, consistent with the long cash-flow duration of the stock market, and it can be used to understand the pricing of the cross-section of equities. Our framework allows us to revisit outstanding questions on why stock prices move, the size of the ex ante equity premium, the impact of interest rates on wealth inequality, and the channels through which monetary policy influences stock prices, all of which we elaborate on below.

The key input for our measurement is an international panel of long-term professional forecasts for interest rates, inflation, and growth rates, which we obtain from Consensus Economics. For each country, we back out forecast-implied series for the trend long-term real rate r^* (a five-year-ahead forecast of the 10-year interest rate, net of expected inflation) and

trend growth rate (real output growth five years ahead), and we augment these with option-based measures of uncertainty. After stripping out growth-rate and uncertainty changes, the remaining interest-rate change is our estimate of the pure discounting shock.

In the U.S., we attribute around 35% of the decline in r^* since 1990 to pure discount-rate changes, and the remaining 65% to the other components. So while equities have benefited somewhat from the decline in U.S. interest rates, assuming full pass-through of r^* to equity yields would overstate the effect by close to three times. And the pass-through of the decline in rates to equities has been even lower in most other G7 countries. In the U.S., the pure discounting term declined substantially in the 1990s; since 2000, it has stayed mostly flat, implying that the decline in rates since 2000 is driven by forces other than pure discounting.

Our measurement of r^* from forecasts of *long-term* bond yields is motivated by a key theoretical result: we show that for non-parallel shifts in the discount-rate curve, the most relevant rate for equity is a long-horizon one. Given this long-horizon measurement, however, one natural concern is that our estimated pure discounting term might spuriously pick up a decline in expected bond term premia. Two features of our results make this interpretation unlikely. First, a term premium change should not in general transmit to equity yields. The near one-for-one pass-through we estimate is therefore difficult to square with a strong role for term premia. Second, we obtain nearly identical results when we measure r^* from short-term, rather than long-term, natural-rate estimates.

In addition to explaining secular changes, our decomposition also speaks to higher-frequency movements in interest rates and equities. Without adjusting for the endogeneity of rates, the raw relation between market returns and yield changes is small and imprecisely estimated. But when we implement our decomposition, pure discount-rate shocks generate strong negative comovement between Δr_t^* and annual equity returns. The loading of stock returns on pure discounting shocks provides a theoretically well-founded measure of equity duration (i.e., the weighted-average time to maturity of cash flows), and we estimate a lower bound for duration of about 20 years for the U.S. market. By contrast, equity returns have a small and insignificant relation to the rate change attributable to growth shocks, and a positive relation with the change attributable to uncertainty shocks. These offsetting components illustrate why equity duration is not equivalent to the price sensitivity to arbitrary changes in rates, and why one must isolate the pure discounting component to estimate duration.

We next use our decomposition to better understand the cross-section of stocks and their exposure to interest rates. Following [Gormsen and Lazarus \(2023\)](#), we sort firms by their predicted cash-flow duration and measure these duration-sorted portfolios' returns. We show that these portfolios do not differ in their exposure to raw interest-rate changes, but that the long-duration firms have significantly greater exposure to the pure discounting shock. This

holds in spite of the unconditional negative alpha to long-duration relative to short-duration stocks, and it implies a sizable spread (greater than 20 years) in the duration of long- versus short-duration firms' cash flows. These results provide a further out-of-sample validation of both the duration sort and the construction of the pure discounting term.

Our results are robust across a range of alternative specifications and measurement choices. This includes the use of alternative measures of r^* , g^* , and uncertainty in our interest-rate decomposition. Moreover, we provide a set of theoretical and empirical robustness results on the role of changes in the profit share of income, or the ratio of earnings to aggregate output. Using separate forecast data on earnings and dividends, we find that our main U.S. empirical results are effectively unchanged when allowing for a time-varying profit share.

Taken together, our results show that isolating the pure discounting term is essential for understanding how shocks to interest rates are reflected in stock prices. While changes to this term are equivalent to changes in the pure rate of time preference for the marginal investor, we do not view this as the only (or main) source of likely variation. To better interpret this term, we discuss how capital flows represent a candidate source of this pure discounting variation, as they may affect interest rates in a manner not fully accounted for by changes in fundamentals (growth or risk). We then show that the changes in the estimated pure discounting terms align reasonably well with cross-country capital flows empirically. We also briefly discuss other plausible contributors to changes in the pure discount rate.

Implications

Our findings on interest-rate pass-through speak to a range of issues raised in the literature:

- (i) Our results clarify analyses of the relation between interest rates and equity valuations. A prominent interpretation of classic valuation decompositions is that valuations move mainly with risk premia rather than interest rates (e.g., [Campbell and Cochrane 1999](#); [Lettau and Ludvigson 2001](#)). In contrast to this “risk premium view,” we find that changes in the pure discounting term explain most of the secular changes in equity yields across G7 countries since 1990, as well as 77% of the time variation in U.S. equity yields in this sample. Because interest rates comove with risk premia, isolating the pure discounting term is key for understanding the role of rates in equity valuation fluctuations. Doing so reveals a larger role for rates than previously recognized.
- (ii) Estimating the ex ante equity premium based on realized returns can lead to biased estimates if interest rates or risk premia change over the sample ([Fama and French 2002](#)). One approach to control for declining interest rates is to subtract from stock returns the returns on duration-matched bond portfolios ([van Binsbergen 2024](#); [Polk and Vuolteenaho 2026](#)). While this is useful for measuring the realized returns associated

with dividend risk, we show that it leads to less precise equity-premium estimates in general. This arises because of the imperfect pass-through from bonds to stocks: only the pure discounting term passes through to equities in line with their duration, and much of the decline in rates reflects growth and risk shocks. We instead suggest subtracting the returns on a duration-matched “pure discounting” claim, which hedges only against pure discounting changes. In simulations, this produces equity-premium estimates that are 20% more precise than comparing stock returns to ex ante yields, and 50% more precise than comparing to duration-matched bond returns. Empirically, we estimate a large ex ante equity premium using our approach.

- (iii) Recent work has argued that the decline in interest rates has influenced wealth inequality. If r^* declines have passed through fully to equity, held disproportionately by wealthy households, then much of the increase in inequality in recent decades may be driven purely by interest-rate changes; see [Fagereng et al. \(2025\)](#) for a review of this literature. Our findings help speak to this debate. We find that only a small share of the decline in r^* was transmitted directly to equity, implying that declining interest rates had a much smaller impact on wealth inequality than calculations based on full pass-through would imply. This argument is consistent with [Greenwald, Lettau, and Ludvigson \(2025\)](#) and [Irie \(2025\)](#), who find a larger role for other forces (particularly cash-flow changes) versus discount-rate declines in explaining equity returns and changes in inequality.
- (iv) Both academic research and practitioner commentary have discussed how the decline in interest rates may have affected factor returns and cross-sectional anomalies (e.g., [Asness 2022](#)). One view holds that portfolios tilted toward short-duration cash flows (such as value, or high book-to-market, portfolios) would have performed better in recent decades in a counterfactual without the rate decline.¹ We find that while value stocks have limited exposure to raw interest-rate movements in general, they are highly exposed to the pure discounting component of rates. We use this distinction to clarify the role of declining rates in value’s recent poor performance: the pure discounting component explains some, though far from all, of this underperformance.
- (v) As a further application, we use our decomposition to unpack the effects of surprise changes in short-term interest rates by monetary policymakers. Some papers implicitly treat the resulting changes in long-term yields as pure discounting shocks, but this need not hold: while the change in the short-term rate is indeed exogenous, the long-term yield responds to changes in both the pure discount rate *and* perceived long-run growth and uncertainty (e.g., [Hanson and Stein 2015](#)). Using our estimation results along with high-frequency asset-price changes, we back out announcement-specific changes in both

¹See [Maloney and Moskowitz \(2021\)](#), and citations therein, for related discussion.

the pure discounting term and expected growth. On average, we find that most of the change in long-term yields around policy shocks indeed stems from the pure discounting component. But we find as well that growth expectations change in a manner consistent with an information effect on average (Nakamura and Steinsson 2018; Gilchrist, Yang, and Zhao 2024), with substantial heterogeneity across announcements.

Overall, we provide a toolkit for understanding how changes in interest rates have, and have not, affected a range of risky assets and aggregate outcomes over the short and long run.

Additional Related Literature

In addition to the connections described above, our paper relates to a long literature on the time-varying relationship between stocks and bonds; Campbell, Pflueger, and Viceira (2025) provide a recent review. Much of this work studies higher-frequency comovement between stocks and *nominal* bonds and the role of inflation risk (e.g., Song 2017; Campbell, Pflueger, and Viceira 2020). We focus on real yields. Chernov, Lochstoer, and Song (2025) study a real channel driven by the mix of permanent and transitory consumption shocks, while Laarits (2025) emphasizes changing precautionary savings motives. Our framework accommodates both of these channels via the uncertainty (i.e., entropy) term in our real-rate decomposition, though we do not focus directly on variation in the high-frequency comovement.

Our paper is closer to work studying longer-term stock–bond comovement. Campbell and Ammer (1993) use VARs to relate this comovement to long-term expected returns. We instead use surveys and asset prices to map bond and equity valuations to three structural drivers of real rates, not to expected returns in reduced form. Barsky (1989) studies the comovement in a two-period framework; we allow for pure discounting shocks and estimate our more general decomposition empirically.² Farhi and Gourio (2018) use a growth model to account for secular valuation changes. Our exercise is less tightly parameterized, but our evidence is consistent with their finding that real rates have comoved negatively with equity premia. Bianchi, Lettau, and Ludvigson (2022) relate equity valuations to the monetary policy stance; this focus is more specific than ours, but the two approaches similarly highlight the need to isolate precise sources of rate variation to understand pass-through to stocks.

We also relate to work studying long-term drivers of stock prices and interest rates. For equity, Greenwald, Lettau, and Ludvigson (2025) argue that increased profit shares are a key driver of stock-price increases in the U.S. data. While their model-based estimation differs from ours, our results are consistent: profit-share increases have affected both prices and cash

²In allowing for pure discounting shocks, we relate to macro–finance work on preference-related changes in discount rates (e.g., Albuquerque et al. 2016). Lettau and Wachter (2011) also study a reduced-form stochastic discount factor and exogenous (pure) real-rate shocks. Our general decomposition also builds on the SDF characterizations of Hansen (2012), Backus, Chernov, and Zin (2014), and Martin (2017).

flows while having a limited effect on their ratio (Section 4.2). For rates, our paper connects to the secular decline in the natural rate (Del Negro et al. 2019; Bauer and Rudebusch 2020). We decompose this trend into structural components and measure how each transmits to equity (and at which horizon). A related literature studies its sources, including demographics, inequality, and global imbalances (Mian, Straub, and Sufi 2021; Auclert et al. 2025; Caballero, Farhi, and Gourinchas 2008). We view these as potential channels driving the components of the real rate, and we show in Section 5.1 that the capital flows emphasized by Caballero, Farhi, and Gourinchas (2008) line up with our estimated pure discount-rate changes.³

Organization

We begin with our theoretical decompositions in Section 2. We then turn to our data, measurement approach, and main findings in Section 3, and Section 4 provides robustness results. In Section 5, we analyze additional implications of our findings. Section 6 discusses and concludes. Derivations and additional results can be found in the Online Appendix.

2. Theory: Rate Components and Equity Valuations

This section develops our theory linking interest rates and equity valuations. We decompose the trend real rate into three interpretable and measurable fundamental components, such that one component induces perfect comovement of bonds and stocks. Isolating this component will then allow us to measure the degree to which rate changes transmit to equity valuations. Our analysis also guides the measurement of each object in the data; we show, for instance, that the trend *long-term* real rate is the relevant starting point for understanding pass-through.

We begin in Section 2.1 with a general real-rate decomposition, imposing minimal assumptions on fundamentals or the stochastic discount factor. We then specialize to an interpretable consumption-based version. We consider what each term in the decomposition means for equity prices (Section 2.2) and for equity duration (Section 2.3), and we then characterize the relevant empirical objects for our estimation (Section 2.4).

2.1 Decomposing the Trend Real Rate

A General SDF-Based Version

We start with a general stochastic discount factor (SDF) M_{t+1} such that $\mathbb{E}_t[M_{t+1}R_{t+1}] = 1$ for an arbitrary asset's gross return R_{t+1} . This implies $R_{t+1}^f = 1/\mathbb{E}_t[M_{t+1}]$, where R_{t+1}^f is the

³More broadly, our joint use of bonds and stocks relates to work using assets' comovements to study underlying shocks, especially bonds and exchange rates (e.g., Kerkre and Lenel 2024). Our use of long-horizon surveys to link fundamentals and returns also parallels Kremens, Martin, and Varela (2025).

real risk-free rate. Taking logs (and denoting logged variables in lowercase),

$$r_{t+1}^f = -\mathbb{E}_t[m_{t+1}] - L_t(M_{t+1}), \quad (1)$$

where $L_t(M_{t+1}) \equiv \log \mathbb{E}_t[M_{t+1}] - \mathbb{E}_t[m_{t+1}]$ is the conditional entropy of the SDF. We note that all expectations $\mathbb{E}_t[\cdot]$ are interpretable as either objective or subjective.

For now, we put very little structure on the SDF. We assume that the log SDF can be additively decomposed as follows:

$$m_{t+1} = \underbrace{-\rho_t}_{\text{predetermined trend}} - \underbrace{(f(X_{t+1}) - f(X_t))}_{\text{difference for Markov } X} + \underbrace{\varepsilon_{t+1}}_{\text{mean 0 martingale diff.}}. \quad (2)$$

This additive representation is constructed following Hansen (2012, Theorems 3.1–3.2), and it holds under a general set of primitive assumptions. See Appendix A.1 for formal details. As discussed there, the term $f(X_{t+1}) - f(X_t)$ is either stationary or difference-stationary.

In interpreting (2), the trend $-\rho_t$ shifts the intertemporal marginal rate of substitution m_{t+1} in all states, so ρ_t can be thought of as a time discount rate. We interpret the Markov state X_{t+1} as determining aggregate cash flows (again see Appendix A.1), so $f(X_{t+1}) - f(X_t)$ can be thought of as the realized marginal utility from cash flow growth. Finally, ε_{t+1} is the remaining martingale component of the log SDF. These terms' interpretation will map to their interpretation in the consumption-based framework in the next subsection.

Plugging (2) into (1), the log risk-free rate satisfies

$$r_{t+1}^f = \underbrace{\rho_t}_{\text{trend (discounting)}} + \underbrace{\mathbb{E}_t[f(X_{t+1}) - f(X_t)]}_{\text{expected growth}} - \underbrace{L_t(M_{t+1})}_{\text{uncertainty/prec. savings}}. \quad (3)$$

The first two terms' labels align with the interpretations discussed above. For the labeling of $L_t(M_{t+1})$ as an uncertainty or precautionary savings term, note that by definition of entropy,

$$L_t(M_{t+1}) = \sum_{n=2}^{\infty} \frac{\kappa_{n,t}(m_{t+1})}{n!}, \quad (4)$$

where $\kappa_{n,t}(m_{t+1})$ is the n^{th} conditional cumulant of the log SDF distribution. Conditional entropy therefore encodes the higher ($n \geq 2$) moments of marginal utility, as is standard.

Our main interest will be in understanding changes in the trend real rate r_t^* . Analogous to Bauer and Rudebusch (2020), we define this as the Beveridge–Nelson permanent component of the one-period rate, $r_t^* \equiv \lim_{s \rightarrow \infty} \mathbb{E}_t[r_{t+s+1}^f]$. But unlike Bauer and Rudebusch, we are interested in longer-horizon real rates: we allow the period length to be long, with one period

potentially standing in for a horizon of many years. (We defer to [Section 2.4](#) the question of which horizon is most relevant for equity.) The zero-coupon yield at arbitrary horizon s is $R_{t,t+s}^f = \mathbb{E}_t[M_{t,t+s}]^{-\frac{1}{s}}$, where $M_{t,t+s} = M_{t+1} \cdots M_{t+s}$, so (3) carries through when relabeling a single period to span horizon s .⁴ At longer horizons, term premia may affect yields through the multi-period entropy term $L_t(M_{t,t+s})$; see [Backus, Chernov, and Zin \(2014\)](#). We use one-period notation for convenience and return to term premia below.

For the trend real rate r_t^* , equation (3) directly implies that it satisfies

$$r_t^* = \rho_t^* + \tilde{g}_t^* - L_{t,M}^*, \quad (5)$$

where $\rho_t^* = \rho_t$, $\tilde{g}_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[f(X_{t+s+1}) - f(X_{t+s})]$ and $L_{t,M}^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[L_{t+s}(M_{t+s+1})]$. This is the first version of our real-rate decomposition into three terms corresponding to discounting, expected growth, and uncertainty. The analysis in this section shows that such a decomposition can be derived quite generally — up to the issue of interpretation of each of the three terms — starting from an additive decomposition of the log SDF.

A Consumption-Based Version

To put more structure on the decomposition in (5), we now consider a more standard consumption-based framework. We assume an endowment economy in which a representative agent has power utility over consumption,⁵

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta_t^s \frac{C_{t+s}^{1-\gamma}}{1-\gamma}. \quad (6)$$

The time discount factor β_t and corresponding rate of time preference $\rho_t = -\log \beta_t$ are potentially time-varying. We assume that relative risk aversion γ , or the inverse elasticity of intertemporal substitution (EIS), is constant.

Given (6), the log SDF is $m_{t+1} = -\rho_t - \gamma g_{t+1}$, where $g_{t+1} \equiv c_{t+1} - c_t$ is log consumption growth. Plugging this into (1),

$$\begin{aligned} r_{t+1}^f &= \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}) \\ &= \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - \sum_{n=2}^{\infty} \frac{(-\gamma)^n \kappa_{n,t}(g_{t+1})}{n!}, \end{aligned} \quad (7)$$

where the final expression for $L_t(M_{t+1})$ in terms of the growth-rate cumulants $\kappa_{n,t}(g_{t+1})$ is as

⁴More formally, $r_{t,t+s}^f = \rho_t + s^{-1} \mathbb{E}_t[f(X_{t+s}) - f(X_t)] - s^{-1} L_t(M_{t,t+s})$.

⁵See [Appendices A.1–A.2](#) for extensions with Epstein–Zin preferences, time-varying risk aversion, or other departures. Decompositions of the form (5) or (8) still hold in these alternative specifications.

in [Backus, Chernov, and Martin \(2011\)](#) or [Martin \(2013\)](#); see [Appendix A.2](#). In a lognormal setting, this simplifies to the familiar solution $r_{t+1}^f = \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - \frac{\gamma^2}{2} \text{Var}_t(g_{t+1})$.

Given (7), the trend real rate can be expressed as

$$r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*, \quad (8)$$

where $\rho_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[\rho_{t+s}]$, $g_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[g_{t+s+1}]$, and $L_{t,M}^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[L_{t+s}(M_{t+s+1})]$.⁶ Thus exactly as in (5), the real rate can move due to changes in (i) time preference (a stand-in for pure discounting shocks), (ii) expected growth rates (via an intertemporal substitution channel), or (iii) risk or uncertainty (via a precautionary savings channel).

2.2 Pass-Through to Equity Valuations

We now move to equity and ask how each of the three terms in our r^* decomposition transmits to equity valuations. This requires further structure on equity cash flows and other fundamentals. Using a set of standard assumptions, we derive a version of a Gordon growth formula for equity dividend yields. We then apply this to show how each component of our real-rate decomposition maps to equity valuations in a simple, intuitive way.

We pick up from the same consumption-based framework as in [Section 2.1](#), though the main equity-valuation decompositions below in fact apply as well under Epstein–Zin utility (see [Appendix A.3](#)). For equity cash flows, we follow [Campbell \(1986\)](#) and [Abel \(1999\)](#) and model equity as a levered claim to consumption, paying dividends $D_t = C_t^\lambda$, with $\lambda > 0$. This imposes a tight link between dividend growth and output and consumption growth. While this works well to explain the long-term trends observed in our data, this need not always be the case (particularly at high frequencies). We accordingly extend our analysis in [Section 4.2](#) to allow for time variation in the profit share of output, which we then discipline in the data with additional survey expectations. For now, however, we assume that λ is constant.

Denote the gross equity return by R_{t+1}^{mkt} and the log return by r_{t+1}^{mkt} , and define $\mu_t \equiv \mathbb{E}_t[r_{t+1}^{\text{mkt}}]$ and $rp_t \equiv \mu_t - r_{t+1}^f$. The equity yield is defined as

$$ey_t \equiv \log(1 + D_t/P_t), \quad (9)$$

where P_t is the price of the equity claim. This is slightly different from the usual log dividend–price ratio ($dp_t \equiv \log(D_t/P_t)$). We define ey_t as in (9) because it puts the equity yield in equivalent units as the log real rate. It also yields straightforward characterizations of steady-state ey_t^* building on results from [Martin \(2013\)](#) and [Gao and Martin \(2021\)](#).

⁶As discussed in [Appendix A.1.3](#), in a stationary setting in which the limiting expectations here are constant, we redefine these terms as infinite-horizon discounted sums of expected outcomes.

To show that our results apply across a range of standard environments, we consider two cases for the dynamics of fundamentals. Each will inform different aspects of the empirical analysis. Case I features permanent shocks to fundamentals. This case gives an exact decomposition for equity yields and is useful for considering secular changes over our full sample, as well as higher-frequency permanent shocks. In Case II, fundamentals are instead stationary. We obtain the same decomposition for equity yields in an approximate (Campbell–Shiller) form, but this setting further allows us to characterize precisely which interest rate is most relevant for measuring the effect of persistent, mean-reverting shocks.

Case I (Gordon Growth): Assume that log output growth $g_{t+1} = c_{t+1} - c_t$ is conditionally i.i.d. with arbitrary distribution, so that as of time t , agents expect growth to be i.i.d. with the current parameters forever. We allow for unanticipated shocks to these parameters, which then become the new steady state. Given this structure, we write $g_t^* = \mathbb{E}_t[g_{t+1}] = \mathbb{E}_t[g_{t+s}]$ for all $s \geq 1$, and the growth-rate cumulants satisfy $\kappa_{n,t}(g_{t+1}) = \kappa_{n,t+s}(g_{t+s+1})$ for all $s \geq 1$. We similarly allow for unanticipated shocks to the preference parameter $\rho_t^* = \rho_t$. This setting provides a clean starting point for understanding secular changes, and we show below that it can be generalized to non-i.i.d. cases without loss of generality.

This i.i.d. environment closely builds on that of [Martin \(2013\)](#), and we apply and extend his results; see [Appendix A.3](#) for details and derivations. Given the constant growth rates and discount rates, a Gordon growth formula applies as follows (where we use the i.i.d. assumption to set all relevant variables equal to their conditional steady-state values):

$$ey_t^* = r_t^* + rp_t^* - \lambda g_t^*. \quad (10)$$

The log equity premium satisfies

$$\begin{aligned} rp_t^* &= \sum_{n=2}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} ((-\gamma)^n - (\lambda - \gamma)^n) \\ &= L_t(M_{t+1}) - L_t(M_{t+1}R_{t+1}^{\text{mkt}}) = L_{t,M}^* - L_{t,MR}^*. \end{aligned} \quad (11)$$

As discussed in the appendix, the fact that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1}R_{t+1}^{\text{mkt}})$ is fully general: it holds under no arbitrage and does not require i.i.d. fundamentals or any assumptions on utility. If $\lambda = \gamma$, then $M_{t+1}R_{t+1}^{\text{mkt}} = 1$, and $rp_t^* = L_{t,M}^*$.⁷ Alternatively, for arbitrary λ and γ , if growth is lognormal so that $\kappa_{n,t}(g_{t+1}) = 0$ for $n > 2$, then $rp_t^* = \frac{1}{2}\lambda(2\gamma - \lambda)\text{Var}_t(g_{t+1}) = \frac{\lambda(2\gamma - \lambda)}{\gamma^2}L_{t,M}^*$.

Using (8), (10), and (11), we obtain a solution for equity yields summarized along with

⁷This is a restatement of the fact that the [Alvarez and Jermann \(2005\)](#) SDF entropy lower bound holds with equality in the growth-optimal case (i.e., with $M_{t+1}R_{t+1}^{\text{mkt}} = 1$): $L_t(M_{t+1}) = \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - r_{t+1}^f$.

the real risk-free rate in the following result.

RESULT 1. *The steady-state real risk-free rate and equity dividend yield satisfy*

$$\begin{aligned} r_t^* &= \rho_t^* + \gamma g_t^* - L_{t,M}^*, \\ ey_t^* &= \rho_t^* + (\gamma - \lambda)g_t^* + (rp_t^* - L_{t,M}^*) \\ &= \rho_t^* + (\gamma - \lambda)g_t^* - L_{t,MR}^*. \end{aligned}$$

Changes in the risk-free rate can arise due to (i) pure discounting shocks (changes in ρ_t^*), (ii) growth-rate shocks (g_t^*), or (iii) risk (entropy) shocks ($L_{t,M}^*$). Each of the three has different implications for equity valuations:

- (i) **Pure discounting shocks:** Bonds and equity comove perfectly, with ey_t^* increasing by 1 basis point for each 1 basis point increase in r_t^* .
- (ii) **Growth-rate shocks:** Equity yields change by $\frac{\gamma - \lambda}{\gamma}$ per unit increase in r_t^* . If $\gamma = \lambda$ (e.g., with log utility and an unlevered consumption claim), ey_t^* is unaffected by changes in r_t^* induced by growth shocks. If $\lambda > \gamma$, growth shocks induce negative comovement.
- (iii) **Risk shocks:** Equity yields change by $-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1$ per unit increase in r_t^* if $\frac{\partial rp_t^*}{\partial L_{t,M}^*}$ is well-defined. Otherwise, equity yields change on average by $-\beta_L + 1$ per unit increase in r_t^* , where $\beta_L \equiv \frac{\text{Cov}(rp_t^*, L_{t,M}^*)}{\text{Var}(L_{t,M}^*)}$. If $\gamma = \lambda$, then ey_t^* is unaffected by changes in r_t^* induced by risk shocks. If $\beta_L > 1$, risk shocks induce negative comovement.

The key implication of this result is that only the pure discounting channel generates perfect pass-through from interest rates to equity yields, and from bond prices to duration-matched stock prices (as discussed below). The range of past work assuming that the decline in rates has passed through fully to equity valuations — as discussed in [Section 1](#) — has therefore implicitly assumed that the decline in r_t^* has arisen due to such pure discounting shocks. Changes in ρ_t^* can be thought of as capturing, for example, demographic changes, or something akin to a savings glut. We discuss such interpretations further in later sections.

For growth-rate changes, note that the equity yield depends on $r_t^* - \lambda g_t^*$ (our version of $r - g$), so a decline in g_t^* will have roughly offsetting effects: it decreases both discount rates and growth rates.⁸ While we do not impose this, it is common to assume that $\gamma \leq \lambda$ — or, in the Epstein–Zin case, that $\frac{1}{\psi} \leq \lambda$, where ψ is the EIS — so that a decline in growth rates

⁸The ey_t^* decomposition in [Result 1](#) does not separate between discount rates and growth rates in the same manner as a Campbell–Shiller decomposition. Instead, it collects terms such that the g_t^* term, for example, contains both the direct cash-flow effect (λg_t^*) and the discount-rate effect (γg_t^*). This reflects our desire to decompose risk-free discount rates into structural components rather than composite terms.

also decreases equity valuations (corresponding to a higher ey_t^*). This would imply that growth-rate changes generate weakly negative comovement between bonds and stocks.

For changes in risk, there are offsetting effects on the risk-free rate and the risk premium. These changes may approximately offset or may cause stocks to move in the opposite direction of bonds. As $L_{t,M}^*$ loads on all the higher cumulants of the growth distribution as in (7), the stock-price response will depend on the specific parameter change underlying the risk shock. In [Appendix A.3](#), we characterize the bond–stock comovement in three benchmark cases: (i) in a lognormal setting with power utility and $\gamma \neq \lambda$, r_t^* and ey_t^* comove positively given changes in risk, though the pass-through is less than one-for-one if $2\gamma > \lambda$; (ii) in a rare-disasters model as in [Barro \(2006\)](#), if $\gamma < \lambda$, then r_t^* and ey_t^* comove negatively given changes in the average disaster size (or, more generally, given changes in skewness or other odd moments); (iii) with Epstein–Zin utility, if $\gamma > 1$, $\psi > 1$, and $\lambda = 1$, then r_t^* and ey_t^* comove negatively given changes in even moments of the growth distribution (variance, kurtosis, and so on), as in [Martin \(2013\)](#). So while positive risk shocks robustly decrease the risk-free rate and generally induce a muted or negative stock–bond comovement, we treat their precise pass-through to stocks as an open question to be disciplined empirically.

[Result 1](#) is our main decomposition for trend real rates and equity valuations. We now show that it also applies under arbitrary permanent shocks, without imposing conditionally i.i.d. fundamentals. To do so, assume that the distributions of growth rates and preference parameters are such that ey_t follows a martingale, $ey_t = \mathbb{E}_t[ey_{t+1}]$. [Campbell \(2018\)](#) refers to this as a “drifting steady state” model for $ey_t^* = ey_t$, and [Campbell and Thompson \(2008\)](#) show that such a model has success at forecasting medium-to-long-horizon returns. To a first order for ey_{t+1} around its expectation ey_t , we have in this case that $ey_t = \mathbb{E}_t[r_{t+1}^{\text{mkt}} - \lambda g_{t+1}]$ ([Gao and Martin 2021](#); again see the appendix). This implies that

$$ey_t^* = ey_t = \mathbb{E}_t[r_{t+1}^{\text{mkt}} - \lambda g_{t+1}] = \mathbb{E}_t[r_{t+2}^{\text{mkt}} - \lambda g_{t+2}] = \dots = r_t^* + rp_t^* - \lambda g_t^*, \quad (12)$$

assuming the individual limiting values exist as $t + s \rightarrow \infty$. In addition, r_t^* satisfies the same decomposition as in (8). [Result 1](#) therefore holds exactly, and the same takeaways apply.

These steady-state facts can be equivalently stated as applying to one-period-ahead conditional expectations:

$$ey_t = \rho_t + (\gamma - \lambda)\mathbb{E}_t[g_{t+1}] + (rp_t - L_t(M_{t+1})),$$

and similarly for the risk-free rate as in equation (7). These versions are useful for interpreting higher-frequency permanent shocks to rates and prices.

Case II (Stationarity): We now instead assume that ey_t and all fundamentals are stationary, with no unanticipated permanent shocks. To non-trivially characterize the pass-through from interest-rate changes to equity valuations in this case, we must reinterpret all the previous starred terms to capture persistent rather than permanent variation. Concretely, we redefine the starred terms $z_t \in \{r_{t+1}^f, rp_t, \rho_t, g_{t+1}, L_t(\cdot)\}$ as Campbell–Shiller-type discounted sums:

$$z_t^* \equiv (1 - \delta) \sum_{s=0}^{\infty} \delta^s \mathbb{E}_t[z_{t+s}], \quad (13)$$

where $\delta \in (0, 1)$ is a loglinearization term defined in [Appendix A.3](#). Since $(1 - \delta) \sum_{s=0}^{\infty} \delta^s = 1$, (13) defines the starred long-run terms as weighted averages of all future expected outcomes.

Given (13), we again follow [Gao and Martin \(2021\)](#) to obtain the loglinear approximation

$$ey_t^* \equiv ey_t = r_t^* + rp_t^* - \lambda g_t^*,$$

exactly as in (10), where $r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*$ and $rp_t^* = L_{t,M}^* - L_{t,MR}^*$. So given the redefinitions in (13), [Result 1](#) again applies exactly as stated.

The pure discounting term ρ_t^* in this case is a weighted average of the expected path of one-period pure discount rates. The permanent shocks in Case I shift this entire “pure discounting curve” in parallel, whereas mean-reverting shocks shift it non-uniformly across horizons (with an effect summarized by $d\rho_t^*$). We consider the full curve in more detail below.

To summarize, in both cases for fundamentals, our decomposition delivers effectively equivalent results, as characterized in [Result 1](#). Only shocks to the pure discounting component of real rates pass through perfectly to equity valuations (in the form of equity yields). Shocks to the other two components in our decomposition — growth rates and uncertainty — generate ambiguous and possibly negative comovement between rates and equity valuations.

2.3 Implications for Equity Duration

Having analyzed the relation between interest rates and equity yields, we now consider what the decomposition implies for equity duration. We show that equity duration does not correspond to the price sensitivity of equity to an arbitrary change in rates, and the only rate change that leads to an equity price change equal to its duration is a pure discounting shock.

To make this point, we first express equity prices in levels. We consider a constant-growth steady state and drop time subscripts to simplify:

$$\left(\frac{P}{D}\right)^* = \frac{1}{\exp(r^* + rp^* - \lambda g^*) - 1} = \frac{1}{\exp(\mu^* - \lambda g^*) - 1} \approx \frac{1}{\mu^* - \lambda g^*}. \quad (14)$$

Equity cash-flow duration $\mathcal{D}$ is the value-weighted time to maturity of expected future cash flows:

$$\mathcal{D} \equiv \sum_{s=1}^{\infty} s \frac{e^{-s(\mu^*)} \mathbb{E}_t[D_{t+s}]}{P} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} \approx \frac{1}{\mu^* - \lambda g^*}. \quad (15)$$

This measure is equivalent to the equity price sensitivity to the log equity discount rate,

$$-\frac{\partial \log P}{\partial \mu^*} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} = \mathcal{D}, \quad (16)$$

which parallels the usual result for the exposure of bond prices to a shift in the yield curve. More important here, though, is price sensitivity to interest-rate changes arising from each of the terms in our decomposition. Given (14), we have the following result describing how price sensitivity depends on the underlying structural driver of interest-rate changes.

RESULT 2 (Three Interest-Rate Sensitivities). *The sensitivity of stock prices with respect to each of the three terms in the interest-rate decomposition $r^* = \rho^* + \gamma g^* - L_M^*$ is as follows.*

(i) *The interest-rate sensitivity of stock prices with respect to pure discount-rate shocks is*

$$\mathcal{S}_{r(\rho)} \equiv -\frac{\partial \log P}{\partial \rho^*} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} = \mathcal{D}.$$

(ii) *The interest-rate sensitivity of stock prices with respect to growth shocks is*

$$\mathcal{S}_{r(g)} \equiv -\frac{\partial \log P}{\partial (\gamma g^*)} = \left(1 - \frac{\lambda}{\gamma}\right) \mathcal{D} < \mathcal{D}.$$

(iii) *Assuming that $\partial_{rp,L} \equiv \frac{\partial r p^*}{\partial L_M^*}$ is well-defined and positive, the interest-rate sensitivity of stock prices with respect to risk shocks is*

$$\mathcal{S}_{r(L)} \equiv -\frac{\partial \log P}{\partial (-L_M^*)} = (1 - \partial_{rp,L}) \mathcal{D} < \mathcal{D}.$$

Part (i) tells us that price sensitivity to the pure discount rate pins down equity duration in a manner equivalent to price sensitivity to the equity discount rate in (16). The other interest-rate terms do not share this feature: price sensitivity to growth shocks is strictly less than duration, as is price sensitivity to risk shocks under general assumptions. To take a benchmark example, with log utility ($\gamma = 1$) and equity modeled as an unlevered consumption claim ($\lambda = 1$), the price sensitivity of equity to a change in rates due to g^* or L_M^* are both exactly zero. More generally, as long as equities move positively with expected growth and

negatively with respect to risk, the rate sensitivities in parts (ii)–(iii) will both be negative. So only a change in rates induced by a shock to ρ^* moves equities in line with their duration, and equity duration is *not* equivalent to price sensitivity to an arbitrary change in r^* .

The fact that equity duration is equal to price sensitivity to pure discount-rate changes also provides a novel avenue for measuring duration. Measuring duration using realized growth rates, as in (15), requires a long sample. Measurement using price sensitivity to μ^* , as in (16), is challenging given the difficulty in measuring expected equity returns. So if our interest-rate decomposition provides reliable estimates of the pure discount rate term ρ^* over time, then estimating the exposure of equity returns to changes in this parameter allows for clean estimation of duration, both for the aggregate market and for narrower portfolios.

2.4 Which Interest Rate? Non-Parallel Discounting Shifts

An open question in the literature is what interest-rate horizon should be used as a benchmark for equity (see Section 5.2 for an example). In our setting, this question arises in terms of implementation: estimating our decomposition empirically requires taking a stand on the horizon at which to measure interest rates and their components. So far, we have treated each term in the rate decomposition as a single number (ρ^* , g^* , and L_M^*). This works well for considering permanent shocks (as in Case I of Section 2.2): these induce parallel shifts in the real yield curve, so all horizons are equally valid for understanding equity pass-through. But with non-parallel shifts (e.g., in the mean-reverting Case II), equity responds differently to shocks at different horizons. We thus face the question of which horizon's change is most relevant. Since shocks to the pure discounting term ρ^* generate perfect pass-through in Result 1, we focus on the pure discounting curve and define the relevant horizon on that curve as the one that similarly generates perfect pass-through.

Let $\rho^*(s) \equiv \mathbb{E}_t[\rho_{t+s-1}]$ denote the expected one-period pure discount rate applying over period s (so $\rho^*(1) = \rho_t$), and let $\{\rho^*(s)\}$ denote the pure discounting curve.⁹ Again start from a flat steady state in which $\rho^*(s) = \rho^*$ for all s (and similarly for all other expected future variables), and we consider a possibly non-parallel shock $\{d\rho^*(s)\}$. Let $P_s \equiv \mathbb{E}_t[M_{t,t+s}D_{t+s}]$ denote the price of the s -period dividend strip paying the single $t+s$ dividend. As usual, equity is a claim to all of these strips, with price $P = \sum_s P_s$ and strip value weights $w_s = P_s/P$. In the flat steady state, strip value weights are geometric in the horizon s , $w_s = (e^{ey^*} - 1)e^{-s(ey^*)}$, and the mean strip-weighted horizon is equal to equity duration, $\sum_s s w_s = \mathcal{D}$.

The following result characterizes the equity response to the shift $\{d\rho^*(s)\}$.¹⁰ We first

⁹As in Section 2.1, we continue to allow a period to be of arbitrary length. For concreteness of interpretation, however, it is useful here to think of periods as being annual, as we do below.

¹⁰Formally, we characterize the directional (Gâteaux) derivative in the direction of the shift.

consider an arbitrary shift, and then an affine case that can be viewed as a first-order approximation to an arbitrary shift. The affine case identifies a single relevant horizon.

RESULT 3 (Non-Parallel Discounting Shifts and the Relevant Horizon). *Start from the flat steady state, and consider an unanticipated shift $\{d\rho^*(s)\}$.*

- (i) **General shift:** *To first order, the equity yield moves by the strip-weighted average of the pure-discounting changes, and the log stock price sensitivity is $\mathcal{D}$ times that average:*

$$dey^* = \sum_{s=1}^{\infty} w_s d\rho^*(s), \quad d\log P = -\mathcal{D} dey^*. \quad (17)$$

- (ii) **Affine shift and the relevant horizon:** *For an affine shift $d\rho^*(s) = a + bs$, the strip-weighted average change satisfies $\sum_{s=1}^{\infty} w_s d\rho^*(s) = d\rho^*(\mathcal{D})$. As a result, the equity yield moves one-for-one with the pure discounting change at horizon $\mathcal{D}$, and the log stock price sensitivity to $\rho^*(\mathcal{D})$ is equal to $\mathcal{D}$:*

$$dey^* = d\rho^*(\mathcal{D}), \quad d\log P = -\mathcal{D} d\rho^*(\mathcal{D}). \quad (18)$$

The relevant horizon is thus equal to equity duration.

- (iii) **Parallel shift:** *If $d\rho^*(s) = d\rho^*$ for all s , then $dey^* = d\rho^*$ and $d\log P = -\mathcal{D} d\rho^*$, recovering [Result 1\(i\)](#) and [Result 2\(i\)](#); every horizon is then equally relevant.*

Part (i) of the result emphasizes that equity in general responds to a weighted average of forward pure discount rates. Part (ii) then tells us that the single horizon that best approximates the overall average change is equal to equity duration.¹¹ As a result, to a first order, ey^* moves one-for-one with $\rho^*(\mathcal{D})$, the pure discount rate at horizon $\mathcal{D}$. By contrast, a shift in the short-term (one-period) pure discount rate $\rho^*(1)$ moves the equity yield by only $w_1 = 1/\mathcal{D}$. To understand equity valuation changes in response to rate shocks induced by ρ^* , then, we must consider longer horizons approximately equal to equity duration.

In the data, we will generally consider measures of expected pure discount rates averaged over future periods m to n : $\bar{\rho}^*(m, n) \equiv \frac{1}{n-m+1} \sum_{s=m}^n \rho^*(s)$. Part (ii) implies that the equity yield moves one-for-one with $\bar{\rho}^*(m, n)$ if and only if its midpoint is equal to duration, $\frac{m+n}{2} = \mathcal{D}$. We therefore aim to estimate pure discount rates starting from long-horizon measures of r^* with mean horizon of approximately $\mathcal{D}$.

Finally, part (iii) formally restates the fact that the horizon of measurement should not matter for permanent shifts in ρ^* . We will test whether this holds for the full-sample secular

¹¹To see that the affine shift first-order approximates a general shift, expand the value-weighted average around $\mathcal{D}$: $\sum_s w_s d\rho^*(s) = d\rho^*(\mathcal{D}) + \frac{1}{2} d\rho^{*''}(\mathcal{D}) \mathcal{D}(\mathcal{D}-1) + \dots$. An affine shift produces $\sum_s w_s d\rho^*(s) = d\rho^*(\mathcal{D})$.

changes we estimate in our data. For higher-frequency changes, we expect non-parallel shifts in the discounting curve to be relevant.

The takeaway is that long-horizon rate measures are most relevant as a benchmark for equity. This conclusion holds even though long-term yields are affected by term premia. As in [Section 2.1](#), term premia affect yields through the entropy term: the average s -period term premium is $L_t^*(M_{t,t+1}) - s^{-1}L_t^*(M_{t,t+s})$. Since the term premium is part of the uncertainty term rather than the pure discount rate, a term-premium change should not transmit directly to equity valuations. A strong estimated pass-through of ρ^* to equity is thus inconsistent with confounding from term premia or similar issues. We turn now to this estimation.

3. Empirical Implementation

We now implement our real-rate decomposition empirically and study how interest rates and their three components transmit to equities across countries. We lay out our data and measurement approach in [Section 3.1](#). We then estimate the terms in our decomposition, first in levels to study secular trends ([Section 3.2](#)), and then in changes to study transmission to equity returns ([Section 3.3](#)) and portfolio returns in the cross-section of stocks ([Section 3.4](#)).

3.1 Data and Baseline Measurement Approach

Recall that our goal is to measure each of the terms in the trend real-rate decomposition from [Result 1](#) (or equation (5)): $r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*$, where ρ_t^* is the pure discounting term, g_t^* is long-term expected output growth, and $L_{t,M}^*$ is uncertainty (entropy). Our approach will be to measure r_t^* and g_t^* as directly as possible; measure $L_{t,M}^*$ using a proxy from option prices; and then back out the pure discounting term as a residual.

For trend long-horizon real rates and expected growth rates, our main input is a panel of long-term forecast data obtained from Consensus Economics. They collect survey expectations of country-level economic and financial outcomes from professional forecasters, with 10–30 forecasters per survey for each country. We use the Long-Term Economic Forecasts data, which are available for the G7 countries (Canada, France, Germany, Italy, Japan, the U.K., and the U.S.) from 1990 through 2023, and for a subset of other developed economies (Netherlands, Norway, Spain, Sweden, and Switzerland) starting in 1995 or 1998. These long-term forecasts are available twice annually for 1990–2013, and quarterly since 2014.

Three key features of the Consensus data are useful here. First, data are available for a panel of countries. Second, the forecasts are provided as forward expectations: we observe separate forecasts for year $t + 1$, year $t + 2$, $\dots$, as opposed to forecasts stated as averages over the next X (e.g., 5 or 10) years. The long-horizon forward forecasts remove cyclical variation

affecting the short-horizon or averaged forecasts; we find this is important for measuring trend variables, as we observe lower volatility and mean reversion than in other forecast data (e.g., SPF or IBES). Third, the forecasters are typically professional economists at large investment banks and firms, whose expectations are likely relevant for asset-price determination.

For all series, we use consensus (mean) forecasts at the five-year forward horizon.¹² To estimate the long-term real rate $r_{t,j}^*$ for date t and country j , we take the consensus forecast of the 10-year nominal interest rate at the end of year $t + 5$ and subtract the inflation forecast for year $t + 5$. We use the forecast of the 10-year yield following [Result 3](#), which shows that the relevant rate for equity is a long-horizon one on the order of equity duration. For the expected growth rate $g_{t,j}^*$, we use the forecast of real output growth for year $t + 5$. One potential concern is a possible mechanical relation between this and $r_{t,j}^*$ arising from forecasters using a model tying these variables together. While this is possible, the fact that these forecasters work at institutions with a key role in trading and pricing assets means that their expectations are relevant irrespective of how they are formed. Further, our main exercises — testing if the estimated real-rate components transmit to equity in the manner predicted by theory — will provide an out-of-sample validation of the rate decomposition. We also consider many alternative measures of $r_{t,j}^*$ and $g_{t,j}^*$ in robustness checks in [Section 4](#).

To proxy for the uncertainty term $L_{t,M,j}^*$, we build on results from [Section 2.2](#) and [Martin \(2017\)](#). We show in [Appendix A.3](#) that if the market is growth-optimal and the distribution of log growth is symmetric, then the entropy of the SDF is equal to that of the market, $L_{t,M,j}^* = L_{t,R,j}^*$. And as shown in [Martin \(2017, Result 3\)](#), $VIX_{t,j}^2$ is proportional to the market’s risk-neutral entropy. We therefore set $L_{t,M,j}^* \propto VIX_{t,j}^2$ for our baseline, with the constant of proportionality estimated in our regressions below. We will also use $VIX_{t,j}^2$ for the risk term in the equity yield decomposition, as would hold in the growth-optimal case or with lognormality. Our VIX-based proxy for risk may induce some mismeasurement at higher frequencies, but less so when examining full-sample secular changes. To measure $VIX_{t,j}^2$, we use a global panel of index option prices from OptionMetrics. The sample, data filters, and calculations are taken from [Gandhi, Gormsen, and Lazarus \(2025\)](#); see [Appendix B.1](#) for details. We calculate the squared VIX at the six-month horizon, which is longer than the CBOE’s 30-day VIX given our desire to measure long-term uncertainty.¹³

As mentioned above, we also consider a range of alternative variables to test the robustness of our decomposition results. This includes using trend short-term (rather than 10-year)

¹²This is the longest horizon for which we directly observe a specific forward forecast. The surveys also elicit average expectations over years 6–10, which give nearly identical results ([Section 4.1](#)).

¹³Illiquidity in long-term options makes it infeasible to calculate something closer to a five-year VIX. [Gandhi, Gormsen, and Lazarus \(2025\)](#) show that implied volatility decays slowly at long maturities, so we view our measure as a reasonable proxy for long-term uncertainty (and again see [Section 4.1](#)).

interest rates for $r_{t,j}^*$; alternatives to Consensus survey data; and alternatives to $VIX_{t,j}^2$ for uncertainty. We describe these alternatives in [Section 4](#), and results are robust in all cases.

For equities, we use data from the CRSP/XpressFeed global database to obtain market indices for each country. To measure equity yields $ey_{t,j}$, we start with the five-year earnings-to-price ratio $\bar{E}_{t-4,t,j}/P_{t,j} = [(E_{t-4,j} + \dots + E_{t,j})/5]/P_{t,j}$, where earnings and prices are calculated on a value-weighted basis for all available stocks in the country (see [Appendix B.1](#)). All equity values are thus measured as aggregates rather than on a per-share basis. We map the earnings yield to the equity yield considered in our theory by multiplying this by 0.5, which is the full-sample average payout ratio: half of earnings (more precisely, 49.4%) are paid out to shareholders in an average year–country observation, with the remainder reinvested.¹⁴

For our cross-sectional analyses, we use returns on duration-sorted portfolios following [Gormsen and Lazarus \(2023\)](#). That paper measures duration based on analyst forecasts of long-term expected earnings growth, or LTG (with higher cash-flow growth indicating a longer duration). We also obtain data on value-sorted portfolios via Ken French’s website.

3.2 Secular Trends

To study long-term trends, we start by estimating our decomposition for trend real rates in levels. For all available dates t and countries j , we estimate a regression

$$r_{t,j}^* = \rho_0 + \gamma g_{t,j}^* + \beta VIX_{t,j}^2 + \Gamma_j + \varepsilon_{t,j}, \quad (19)$$

with country fixed effects Γ_j . In our main specification, we also allow the VIX^2 loading β to differ by country (β_j). While this is not important for our main results, it helps account for cases in which sovereign credit risk affects a country’s r_t^* .¹⁵ It also allows for the possibility of country-specific measurement error in the VIX, which may be an issue particularly for countries with less-liquid option markets.

Given a set of estimated coefficients and OLS residuals, we then back out the implied pure discounting term as

$$\hat{\rho}_{t,j}^* = \hat{\rho}_0 + \hat{\Gamma}_j + \hat{\varepsilon}_{t,j}. \quad (20)$$

We thus have, by construction, that $r_{t,j}^* = \hat{\rho}_{t,j}^* + \hat{\gamma} g_{t,j}^* + \hat{\beta} VIX_{t,j}^2$, which corresponds exactly to our theoretical decomposition (with the uncertainty term $-L_{t,j}^*$ proxied by $\hat{\beta} VIX_{t,j}^2$).

Estimates for the regression (19) are shown in [Table 1](#), first for the U.S. only and then for the full 12-country panel. The estimates correspond well to our theory. The estimated loading

¹⁴An alternative is to use dividend yields $dp_{t,j}$ directly in place of $ey_{t,j}$. Since $\Delta dp_{t,j}$ and $\Delta ey_{t,j}$ are highly correlated, this gives near-identical results: the cross-country regression of $\Delta dp_{t,j}$ on the pure discounting change has a slope of 1.01 and an adjusted R^2 of 0.75, as compared to 1.14 and 0.79 in the right panel of [Figure 1](#). Our baseline starts from an earnings yield to address the fact that payout ratios have declined in

Table 1: Regressions for Trend Real Rates $r_{t,j}^*$

	(1) U.S.	(2) All	(3) All
Expected growth $g_{t,j}^*$	1.8*** (0.2)	2.1*** (0.2)	2.1*** (0.2)
Uncertainty $VIX_{t,j}^2$	-10.1** (4.5)	-3.8 (3.0)	β_j
Constant	-1.9*** (0.5)	-1.9*** (0.4)	-2.0*** (0.4)
Country FEs	X	✓	✓
Country-Specific $VIX_{t,j}^2$ Loading	✓	X	✓
Obs.	86	932	932
R^2	0.57	0.65	0.66
Within R^2	—	0.60	0.61

Notes: This table shows OLS coefficient estimates in the regression (19), with standard errors in parentheses. In column (1), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (2)–(3), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. In column (3), the country-specific loadings on the squared VIX, β_j , are statistically significant at the 1% level for 9 of the 12 countries in our sample. The sample is 1990–2023, or the longest available span for the given country.

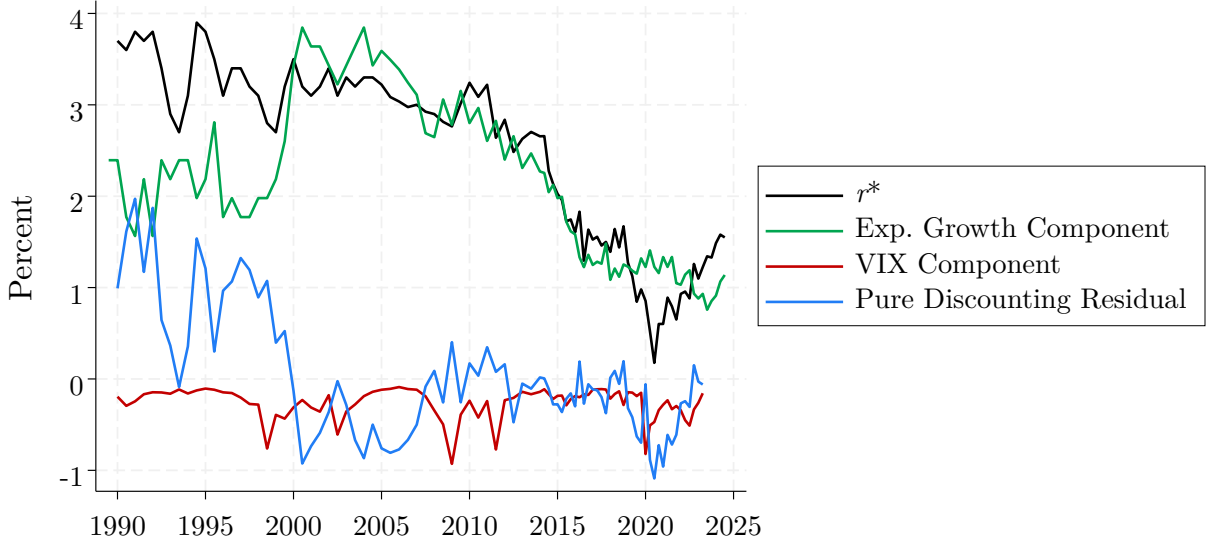
on expected growth is strongly positive and consistently close to a value of 2, corresponding to implied relative risk aversion of $\gamma \approx 2$ and EIS of about $1/2$. The loading on the VIX is negative and significant in the U.S. case and for most countries in the country-specific case shown in column (3). This limited set of variables explains a large share of the variation in trend real rates, with R^2 values of around 0.6 within-country and slightly higher overall. The remaining variation is then attributed to the pure discounting residual.

To visualize the data, Figure 2 presents the estimation results for the decomposition of $r_{t,j}^*$ over time in the U.S. data. The trend real rate has fallen by close to 2.5 percentage points (pp), or 250 basis points (bps), from the beginning to the end of the sample, starting near 4% and ending near 1.5%. As can be seen in the green line, a large share of this decline is attributed to a decline in long-term expected growth. Expected growth fell by around 0.75 pp over the sample, which when multiplied by $\gamma \approx 2$ translates to a predicted decline in yields of about 150 bps. While uncertainty affects real rates significantly during deep recessions, it has only a small negative long-term effect over the full sample. Together, the change in growth rates and uncertainty predicted a decline in real rates of around 160 bps

recent decades. In subsequent higher-frequency time-series plots, we generally use the dividend yield directly.

¹⁵Our theory suggests that the loading on uncertainty should be negative, but credit risk can induce an offsetting positive relation between risk and long-term rates. We find that this effect is small on average.

Figure 2: U.S. Estimation Results for Decomposition of r^* in Levels



Notes: This figure shows the U.S. trend real rate $r_{t,j}^*$ and its components over time, estimated using (19)–(20) following the main specification in column (3) of Table 1. For readability, the expected growth component is shifted down by 3 percentage points ($\hat{g}_{t,j}^* - 3$), and the pure discounting residual is plotted as $\hat{\varepsilon}_{t,j}$.

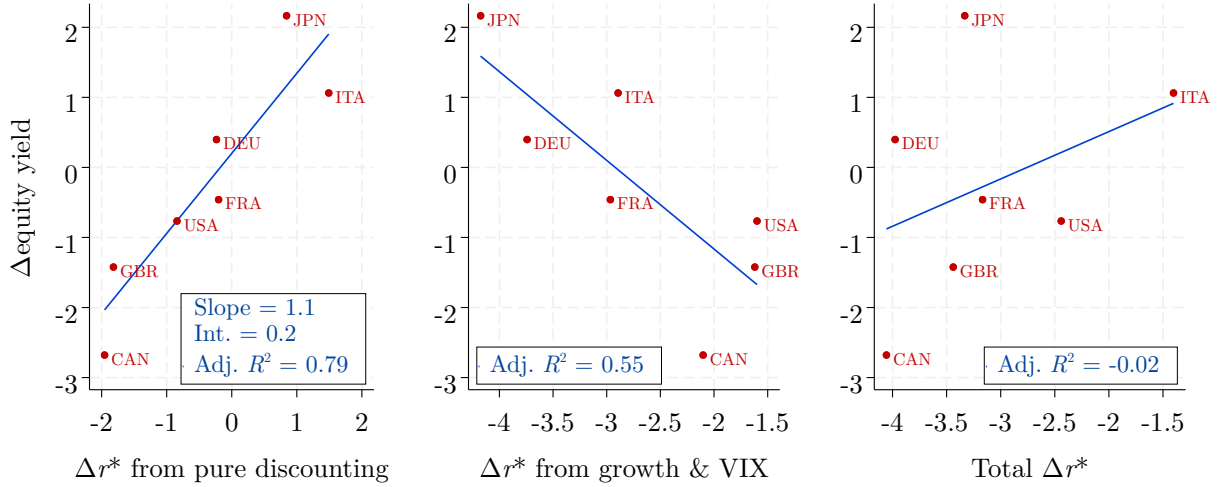
overall, so the additional 90 bps of unexplained decline is attributed to the pure discounting residual. This residual was particularly important in explaining the decline in interest rates early in the sample. From 2000 onward, the decline in interest rates has been driven almost exclusively by declines in expected growth rates, implying little impact on equity valuations. As we show in Section 4, this finding is robust to numerous alternative ways of implementing our $r_{t,j}^*$ decomposition, including the use of shorter-horizon interest rates.

The remainder of this subsection studies how secular changes in equity valuations across countries relate to changes in the different components of interest rates. Our goal is to understand country-level changes in equity valuations over our sample period.

In the leftmost panel in Figure 3, we plot the change in equity yields against changes in the pure discounting term in G7 countries. This is the same figure plotted in the right panel of Figure 1 in the introduction. The figure illustrates that the large majority of the changes in equity yields over this sample can be explained by changes in the pure discounting term in interest rates; we show that this relationship is statistically significant in Section 4.1. Since the pure discounting term is estimated purely from the interest-rate decomposition in (19)–(20), without the use of equity valuations, there is nothing mechanical about the explanatory power of pure discounting changes for stock valuations over this sample. This finding therefore serves as a strong out-of-sample validation of the rate decomposition.

The magnitude of the relation between equity yields and the pure discounting term is,

Figure 3: Main Results: Long-Term Decomposition



Notes: This figure plots the country-level changes in equity yields against changes in different components of interest rates, estimated using (19)–(20) following the main specification in column (3) of Table 1. The leftmost figure plots changes in equity yields against changes in the pure discounting term; the middle figure plots changes in equity yields against changes in the growth and VIX components; the rightmost figure plots changes in equity yields against changes in real rates themselves. The sample is 1990–2023, or the longest available span for the given country. For countries for which we can only measure equity yields starting after 1990 (see Appendix B.1), we calculate both $\Delta \text{equity yield}$ and Δr^* over the same window.

in addition, almost exactly equal to that predicted by theory. The figure shows that equity yields decrease by one percentage point for every one-percentage-point decrease in the pure discounting term, as in Result 1. And the pure discounting term explains not only relative changes in equity valuations across countries, but also changes in valuations in absolute terms: the intercept for the fit is very close to zero, which means the average equity yield has moved by as much as the pure discounting term. This does not necessarily imply that other factors influencing valuation ratios — such as growth rates and risk premia — have remained constant, but it does imply that potential movements in growth rates and risk premia have, on net, not played a significant role in changing equity valuations on average over this period. The upshot is that to understand long-run valuation changes in this sample, understanding the change in pure discount rates is nearly sufficient.

The middle panel of Figure 3 illustrates the relation between equity yields and the change in interest rates induced by changes in expected growth rates and uncertainty, taken together. As expected, we find that valuation ratios have dropped in countries where interest rates have dropped because of declines in growth rates and increases in risk: while these changes have decreased interest rates, they have also depressed growth rates on equities and increased equity premia, with the predicted effect on equity valuations being negative. This relationship is noisier than the one in the left panel, consistent with the more ambiguous theoretical

predictions for valuations given changes in growth rates and uncertainty, but the negative relationship is at least moderately strong in the cross-section of G7 countries.

How can the negative relation in the middle panel be squared with the fact that the pure discounting change can nearly perfectly explain the change in equity valuations over time (as documented in the left panel)? Two aspects of the results help in interpreting this. First, note that the best-fit line in the middle panel does not pass through the origin: unlike the $\Delta\text{equity yield}-\Delta\text{pure discounting}$ relationship in the left panel (which features an intercept indistinguishable from zero), the line in the middle panel is shifted by 2.9 percentage points to the left. Enforcing an intercept of zero in this $\Delta\text{equity yield}-\Delta\hat{r}^*$ relationship, we instead estimate a very small slope (close to -0.1) and an adjusted R^2 of -0.15. Explaining changes in valuations in absolute terms evidently requires using the pure discounting change.

Second, after accounting for the pure discounting change in the left panel, the remaining terms in $\Delta\hat{r}^*$ provide very little additional explanatory power for long-term valuation changes. In a regression for $\Delta\text{equity yield}$ on both the pure discounting change and the remaining $\Delta\hat{r}^*$ terms, only the coefficient on the pure discounting change is significant (slope 0.9, $p < 0.01$), and the adjusted R^2 increases only from 0.79 (in the left panel of Figure 3) to 0.81. To visualize this marginal contribution from growth and uncertainty, Appendix Figure B.1 shows a version of the middle panel of Figure 3 where the change in the equity yield is residualized against the change in the pure discounting term. The part of the equity yield change unexplained by the pure discounting change is generally small quantitatively, and it is now at most very weakly related to the change in rates from the growth and uncertainty terms, consistent with the more ambiguous effects predicted theoretically.

Moving to the right panel of Figure 3: while we observe comovement between equity yields and the different components of interest rates, there is almost no relation between equity yields and rates themselves. The components of rate changes have happened to be somewhat negatively related (albeit weakly so) across countries, so adding the horizontal-axis values in the two left panels of the figure generates a muddled and weak relationship between rates and equity yields. In addition, while the estimated slope of the best-fit line is positive, note that it again does not pass through the origin: the average advanced economy had close to no equity valuation change, while nonetheless experiencing growth-rate and uncertainty shocks large enough to decrease real rates by nearly 300 bps. This emphasizes how comparing equity valuations to real rates directly can paint a misleading picture.

Discussion and Interpretation

Taken together, Figure 3 provides a clear view of both (i) the secular declines in real rates across countries in recent decades, and (ii) their relation to equity valuations. For the U.S., the

part of the r^* decline unexplained by changes in expected growth and uncertainty — around 90 bps, or 35% of the total r^* decline — represents the pure discounting shock, akin to a decrease in the pure rate of time preference. Such a decrease predicts an increase in equity valuations, exactly as we see in the left panel of the figure. Taking Japan as a contrasting case, its decline in r^* of 330 bps is a much smaller decline than would have been expected on the basis of the large decrease in long-term expected growth, indicating a positive pure discounting shock. This positive shock similarly perfectly matches the decrease in Japanese equity valuations. The same applies for all the other countries considered.

While the pure discounting shocks provide a very good description of equity valuation changes in an accounting sense, the question of how to interpret them remains somewhat open thus far. We do not view these changes as likely representing a true aggregate preference (or patience) shock among domestic investors. Instead, a “global imbalances” view of cross-country capital flows, as described by [Caballero, Farhi, and Gourinchas \(2008\)](#), appears to be a reasonable candidate explanation. The main decline in the U.S.’s estimated ρ^* occurred in the mid-to-late 1990s and early 2000s (see [Figure 2](#)). This period coincides with a large decrease in the U.S.’s net foreign asset position. Japan’s estimated ρ^* , meanwhile, increased during this decade. Strong demand for U.S. assets, particularly from investors in countries experiencing large shocks to the perceived soundness of their financial system (e.g., in the wake of the Japanese stock-market crash), matches both the timing and the cross-country patterns observed in [Figure 3](#), as we discuss in greater detail in [Section 5.1](#) below.

3.3 Higher-Frequency Changes and Forecasting Regressions

Interest-rate movements influence not only secular changes in valuations but also higher-frequency fluctuations. In this subsection, we study how stocks move with the different components of real rates at a higher frequency. This allows us to examine these relationships over time within a country, in contrast to considering full-sample changes across countries.

As motivating evidence, in [Figure 4](#), we first examine whether variation in the previously estimated pure discounting term aligns with stock-price changes over time in the U.S. data. The figure illustrates a strong comovement between the two. Regressing the pure discounting term on the equity yield allows us to estimate the share of variation in the equity yield that can be explained by pure discounting, using the slope coefficient in this regression.¹⁶ That number is 77%, emphasizing the important role of the pure discounting term on its own.

This strong comovement arises even though the pure discounting term is estimated without any direct information about equity yields: it uses data only on interest rates,

¹⁶Writing $ey_t = \rho_t + \text{other terms}$, equity yield variance is $\text{Var}(ey_t) = \text{Cov}(\rho_t, ey_t) + \text{Cov}(\text{other terms}, ey_t)$, yielding the decomposition $1 = \frac{\text{Cov}(\rho_t, ey_t)}{\text{Var}(ey_t)} + \frac{\text{Cov}(\text{other terms}, ey_t)}{\text{Var}(ey_t)}$. The first term is the relevant regression slope.

growth expectations, and estimated entropy. One might worry that analysts back out their growth forecasts from equity yield data, and that the growth expectations we use in our decomposition thus mechanically contain information about equity yields. However, such behavior would bias us *against* finding a relation between the pure discounting term and equity yields: the pure discount rate is orthogonal to growth expectations by construction, so if analysts assign variation in equity yields to expected growth, such variation would be mechanically orthogonal to our pure discounting term.¹⁷

Figure 4 uses the pure discounting term estimated from a regression in levels; we now estimate regressions in differences to study relationships between higher-frequency changes. This further sidesteps the potential for spurious comovements between slowly moving variables in levels. We balance two considerations in determining the frequency of measurement. First, we wish to explain price and interest-rate variation for reasonably short holding periods. Second, our estimation needs to allow for inertia in forecasters' long-run growth and interest-rate forecasts, which precludes us from considering, for example, monthly returns (since forecasts are collected at most once per quarter). In our baseline analysis, we consider three-year returns and estimate how these move with each of the components of real rates.¹⁸

The starting point for this analysis is a regression for changes in trend real rates, analogous to equation (19) but in differences:

$$\Delta r_{t,j}^* = \alpha_0 + \gamma \Delta g_{t,j}^* + \beta_j \Delta \text{VIX}_{t,j}^2 + \Gamma_j + \varepsilon_{t,j}, \quad (21)$$

where Δ denotes a three-year change and where the loading on the VIX term is again country-specific.¹⁹ The residual term $\varepsilon_{t,j}$ is now our measure of $\Delta \rho_{t,j}^*$. Next, given this estimated pure discounting change $\widehat{\Delta \rho_{t,j}^*} = \widehat{\varepsilon}_{t,j}$, we regress three-year value-weighted net market returns on that pure discounting term, the change in expected growth, and the change in the VIX, along with a country fixed effect:

$$r_{t,j}^{\text{mkt}} = \alpha_1 + \pi_\rho \widehat{\Delta \rho_{t,j}^*} + \pi_g \Delta g_{t,j}^* + \pi_V \Delta \text{VIX}_{t,j}^2 + \Lambda_j + \nu_{t,j}. \quad (22)$$

¹⁷As an extreme case, imagine that rates depend only on pure discounting and growth, and that analysts back out expected growth from the equity yield ($\mathbb{E}_t^{\text{analysts}}[g_{t+1}] = ey_t + \text{constant}$). We would then estimate

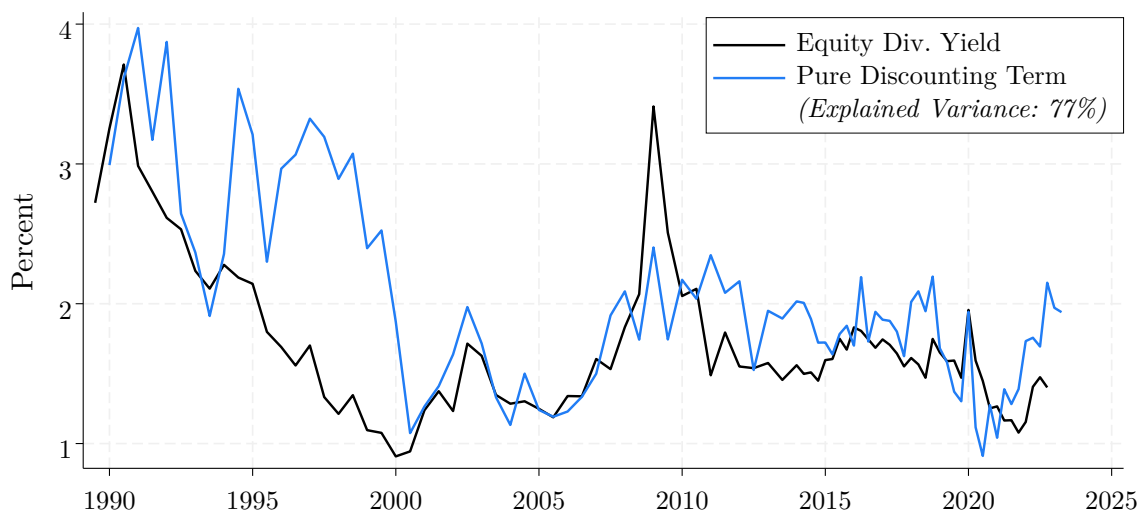
$$r_t^* = \alpha + \gamma \mathbb{E}_t^{\text{analysts}}[g_{t+1}] + \varepsilon_t = \tilde{\alpha} + \gamma ey_t + \varepsilon_t,$$

and recover $\widehat{\rho}_t^* = \tilde{\alpha} + \varepsilon_t$, which would be orthogonal to ey_t by construction.

¹⁸In additional analysis, we find that our results are robust to the use of longer or somewhat shorter windows, though the relationships weakened for windows shorter than two years (indicating inertia or measurement error).

¹⁹Coefficient estimates for regression (21) are presented in Table B.1 of Appendix B.6. The results are similar to the level estimates in Table 1 but with smaller point estimates, suggesting stronger measurement-error attenuation when estimating in differences (Griliches and Hausman 1986).

Figure 4: U.S. Equity Valuations and the Pure Discount Rate Over Time



Notes: This figure shows the U.S. equity dividend yield and the pure discounting term over time, estimated using (19)–(20) following the main specification in column (3) of Table 1. For readability, the pure discounting term is plotted as the pure discounting residual shifted up by 2 percentage points ($\hat{\varepsilon}_{t,j} + 2$).

Table 2 shows the resulting estimates. Before considering our main estimates from (22), we start with a simpler exercise as a benchmark: we regress three-year stock returns on the unadjusted change in the traded 10-year nominal yield. Column (1) shows the resulting estimate for the U.S. sample. The slope coefficient is close to zero and statistically insignificant, reflecting the well-known fact that returns on stocks and bonds are close to uncorrelated.²⁰

In column (2), we present coefficient estimates from (22) in the U.S. data, showing how stock returns load on each of the three drivers of interest-rate changes: changes in the pure discount term, changes in expected growth, and changes in risk. The loading on the pure discount term is -19 and statistically significant. This loading suggests that stock returns go down by 19 percentage points when trend real rates increase by 1 percentage point due to pure discounting. As in Result 2(i), this coefficient has a clear structural interpretation: it is equal to the negative of the cash-flow duration of the overall market. This market-level duration is often approximated by the inverse dividend yield, generating an estimate on the order of 40 years (see, e.g., Gormsen and Lazarus 2023). While somewhat lower than that figure, the estimate of 19 years from column (2) is of the same rough order of magnitude and reinforces that the market is a long-duration claim. We view the estimate as a lower bound given the potential for attenuation bias when using the higher-frequency variation in the pure discounting term (see footnote 19 for related discussion).

The remaining estimates in column (2) show that stock returns load weakly on expected growth changes, and significantly negatively on changes to risk, again consistent with theory.

²⁰When regressing stock returns on the change in our r_t^* , the slope is also small and insignificant.

Table 2: Regressions for Three-Year Stock Returns

	(1) U.S.	(2) U.S.	(3) All	(4) All
$\Delta 10y$ yield	4.19 (3.51)		-3.39 (2.20)	
$\Delta \text{pure discount } (\widehat{\Delta \rho_t^*})$		-19.1** (7.64)		-9.61** (3.26)
$\Delta \text{exp. growth}$		-1.49 (14.0)		16.9* (8.82)
$\Delta \text{VIX}^2 \times 100$		-3.08** (1.33)		-5.44*** (0.90)
Country FEs	X	X	✓	✓
Obs.	74	74	781	781
R^2	0.04	0.20	0.05	0.27
Within R^2	—	—	0.02	0.24

Notes: This table shows estimates from regressing three-year value-weighted market returns on changes in 10-year nominal yields (in columns (1) and (3)), and on changes in the three interest-rate components (in columns (2) and (4)). The interest-rate components are estimated from (21), and the table presents estimates from (22). Columns (1)–(2) consider the U.S. only, while (3)–(4) consider the full panel of developed countries (and include country fixed effects). In columns (1)–(2), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (3)–(4), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The sample is 1990–2023, or the longest available span for the given country.

Moving to columns (3) and (4) of Table 2, the results are largely similar in the global sample. The slope on the pure discounting shock is smaller in the global data than in the U.S., conceivably reflecting greater measurement error in the non-U.S. data. Growth shocks are also estimated to play a somewhat larger role in the global data.

These estimates allow for an accounting of the period-by-period contribution of different interest-rate components to stock returns. We illustrate this higher-frequency return decomposition for the U.S. data in Appendix Figure B.2, which plots the three-year annualized equity return and each of the three fitted components from the return regression. Shocks to risk, growth (including contemporaneous dividend growth), and pure discounting each affect returns significantly in different periods.

As a final out-of-sample validation test for our rate decomposition, we ask whether our estimated pure discounting term $\widehat{\rho}_t^*$ predicts *future* equity returns (in addition to helping account for contemporaneous realized returns). Long-horizon expected equity returns are equal to $\mu_t^* = r_t^* + rp_t^*$. Given that the uncertainty component of r_t^* is likely to be negatively correlated with the equity risk premium rp_t^* , interest rates by themselves are unlikely to

be useful for predicting future realized returns. The pure discounting component of r_t^* , by contrast, strips out the uncertainty component of risk-free rates, and therefore should align well with future equity returns. We conduct such predictability tests in Appendix [Table B.2](#), which shows coefficients from regressions of annualized market returns over the subsequent three years on ex ante yield-related predictors. Neither nominal yields nor our r_t^* help predict equity returns on an unadjusted basis. This provides evidence that risk premia comove negatively with risk-free yields, as emphasized by [Farhi and Gourio \(2018\)](#). Meanwhile, the pure discounting term strongly predicts future returns.

Our rate decomposition therefore succeeds at stripping out shocks to risk-free yields with offsetting effects on equity risk premia or growth rates, leaving us with a useful measure of the pure discounting component of long-term rates. This pure discounting term can be exploited as a counterfactual long-term risk-free rate to use in calculating a duration-matched equity premium; we will return to this insight in [Section 5.2](#).

3.4 Cross-Sectional Portfolios

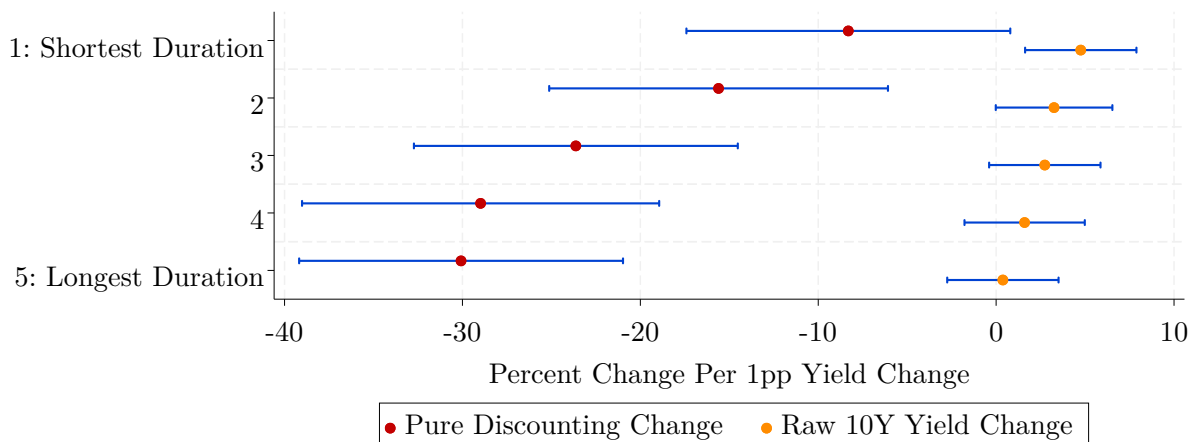
We now turn to the cross-section of stock returns, and study whether firms with different cash-flow duration have different exposure to the pure discounting term. A large literature studies the risk and return properties of firms with different cash-flow timing (see [Gormsen and Lazarus 2023](#), and citations therein), finding that firms with shorter-duration cash flows have higher risk-adjusted returns. In this section, we use our methodology to quantify cross-sectional differences in duration, which is a key (and debated) object for this literature.

We focus here on the measure of duration used in [Gormsen and Lazarus \(2023\)](#), which is based on firm-level cash-flow growth estimates. We first obtain analysts’ median forecasts of stock-level long-term earnings growth (LTG) from IBES (see [Gormsen and Lazarus 2023](#) for details). For each year, we sort stocks into quintile portfolios based on their cross-sectional LTG rank; we do so separately for the U.S. and global data. These are then our five duration-sorted portfolios for which we calculate value-weighted returns. According to this exercise, firms with higher expected cash-flow growth have, all else equal, longer cash-flow duration.²¹

In [Figure 5](#), we report slope coefficients of regressions of three-year realized returns onto three-year changes in the pure discounting term for our five portfolios of U.S. stocks with different cash-flow duration. The figure shows that the portfolio of firms with the shortest cash-flow duration has a slope coefficient of around -10, while the portfolio of firms with the longest cash-flow duration has a slope of -30. At face value, these estimates suggest that the

²¹Another standard approach is to proxy for duration by valuation ratios (like book to market), since a higher valuation is associated with a longer cash-flow duration (i.e., growth firms are long-duration firms). Using book-to-market ratios to measure duration does not affect this section’s results.

Figure 5: Portfolio Exposures to Pure Discount Rates and Yields: U.S. Stocks



Notes: This figure shows slope estimates from univariate regressions for three-year returns of each of five equity portfolios on the three-year change in (1) the pure discounting term ($\Delta \rho_t^*$), marked in red, and (2) the nominal 10-year yield, marked in orange. Each regression contains a constant. For example, the first point in the top left of the figure shows $\hat{\beta}_1$ (and 95% confidence intervals) from $r_{t,1} = \alpha_1 + \beta_1 (\Delta \rho_t^*) + \varepsilon_{t,1}$, where $r_{t,1}$ is the three-year return on a value-weighted portfolio of the stocks in the bottom quintile of cash-flow duration. The pure discounting term is estimated from (21) using U.S. data. Duration-sorted portfolios and returns are calculated following Gormsen and Lazarus (2023). The sample is 1990–2023.

cash-flow duration of these portfolios varies from 10 to 30 years, significant both economically and statistically. We show corresponding results for the global sample in Figure B.3.

As with the previous analysis, it is possible that the slope coefficients suffer from attenuation bias. If such attenuation bias is driven by classical measurement error, it is similar (in percentage terms) for the different portfolios. In this case, it is useful to focus on the ratio of the cash-flow duration of the different portfolios, as this ratio will be unaffected by classical measurement error. We find that the portfolio of firms with the longest duration has three times as long a cash-flow duration as the portfolio of firms with the shortest duration. A lower bound on this difference appears to be 20 years, but we cannot rule out that it is longer.

By contrast, as can be seen in the coefficients plotted in orange, long-duration stocks are not substantially more exposed to raw interest-rate changes than short-duration stocks: all of them have very small estimated loadings when regressing their returns on the change in 10-year nominal yields. And the estimated coefficients go in the “wrong” direction, at least with respect to an interpretation of all interest-rate changes as being exogenous pure-discounting shocks: rather than returns *decreasing* when interest rates increase, they instead weakly increase. This further reinforces the point made in Section 2.3: equity duration does not correspond to the price sensitivity of equity to an arbitrary change in interest rates. Instead, only pure discounting shocks induce interest-rate variation that passes through to equity in proportion to its duration. Duration-sorted portfolios should not, and do not, vary

significantly in their exposure to nominal interest rates by themselves; instead, they vary only in their exposure to pure discount-rate changes.

4. Robustness and Statistical Tests

We now assess the robustness of our main results to alternative measurement choices, and we also provide formal statistical tests of the results in [Figure 3](#). We first show in [Section 4.1](#) that our rate decomposition yields similar results under a range of approaches for measuring the trend real rate and its components and is statistically robust. [Section 4.2](#) then discusses how our analysis generalizes to handle the case in which dividend growth is not proportional to output growth due to time-varying profit shares.

4.1 Testing Alternative Measures for the Rate Decomposition

[Table 3](#) shows a series of robustness tests for our main results under alternative measurement approaches for the real-rate decomposition. The top panel repeats the main cross-country pass-through analysis from the first panel of [Figure 3](#) (and [Figure 1](#) in the introduction), confirming that the one-for-one relationship between equity yields and the pure discounting term holds across specifications. The bottom panel verifies the U.S. time-series properties of ρ^* documented in the previous section.

Column (1) reports results from the baseline specification. For the top panel, this is the same regression as in the first panel of [Figure 3](#), but now with standard errors and statistical significance reported. These are computed via a non-stationary block bootstrap described in [Appendix B.2](#). This bootstrap accounts for estimation uncertainty in the generated regressor $\hat{\rho}^*$, providing statistical evidence to buttress the strong visual relationship in [Figure 3](#). The slope coefficient of 1.14 is highly significant and statistically close to the theoretical benchmark of 1, and the intercept is not significantly different from zero.

The remaining columns vary the inputs to the r^* decomposition, with data described fully in [Appendix B.3](#). Column (2) uses forecasts from the Survey of Professional Forecasters (SPF) for the 10-year real yield and real GDP growth over the next 10 years, in place of the baseline Consensus forecasts. The top panel for this column is blank given that the SPF is U.S.-specific. Column (3) replaces our baseline r^* with the cross-country natural-rate estimates of [Del Negro et al. \(2019\)](#). These are statistical-model-implied estimates of the trend *short-term* real rate, so they differ from our baseline both in maturity and in being model-based rather than survey-based. Because they explicitly separate the trend in expected short rates from the term premium in long-term yields, the estimates are free of any term-premium component. Column (4) uses a country-level 6-month GARCH(1,1) forecast of equity return variance

Table 3: Robustness to Alternative Measures for r^* Decomposition

	(1)	(2)	(3)	(4)	(5)
	Baseline	SPF-Based	Short-Term r^*	GARCH Vol.	Unc. Index
<i>Inputs changed:</i>	—	r^*, g^*	r^*	L^*	L^*
<i>Robustness of Figure 3: Regressions of Δey^* on $\Delta \rho^*$ (G7)</i>					
Slope	1.14*** (0.23)	—	1.01*** (0.16)	1.01*** (0.20)	1.15*** (0.24)
Intercept	0.20 (0.31)	—	0.64** (0.28)	0.31 (0.29)	0.28 (0.30)
Adj. R^2	0.79	—	0.74	0.79	0.79
<i>Robustness of ρ^*: Time-Series Properties (U.S.)</i>					
$\Delta \rho^*$ over sample	-0.84	-1.35	-0.89	-0.94	-0.91
Corr($\rho^*, \rho_{\text{baseline}}^*$)	1.00	0.83	0.84	0.98	0.99

Notes: This table shows robustness of the main results to alternative measurement approaches for the r^* decomposition. All r^* decompositions follow the specification in column (3) of Table 1 for measures available across countries (columns (1), (3)–(5)), and they follow column (1) of Table 1 for the U.S.-only measure (col. (2)). Col. (1) uses the baseline specification as in the left panel of Figure 3. Col. (2) replaces the Consensus data with SPF forecasts for the average 10-year real yield and real GDP growth over the next decade. Col. (3) replaces the Consensus-based r^* with the cross-country short-term natural-rate estimates of Del Negro et al. (2019). Col. (4) replaces VIX² with a country-level 6-month GARCH(1,1) forecast of equity return variance. Col. (5) replaces VIX² with the country-level Baker, Bloom, and Davis (2016) uncertainty index. The top panel shows cross-country (G7) regressions of full-sample changes in equity yields on changes in ρ^* , for all available specifications. Standard errors in parentheses are obtained via block bootstrap by year with 10,000 draws. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The bottom panel shows U.S. time-series results. The first row shows the full-sample change in the estimated ρ^* . The second row shows the correlation between the alternative measure of ρ^* and the baseline measure from column (1). The sample is 1990–2023, or the longest available for the given country.

in place of VIX², to help address the possibility that VIX² may not adequately measure uncertainty or its relation to equity risk premia. Similarly, column (5) uses the country-level Baker, Bloom, and Davis (2016) economic policy uncertainty index in place of VIX².

Across all specifications, the results are strongly consistent. For the cross-country regressions, the slope coefficients range from 1.01 to 1.15, all highly significant and close to the theoretical prediction of one-for-one pass-through. The U.S. time-series results are similarly robust: the full-sample change in ρ^* ranges from -0.84 to -1.35 percentage points across specifications, generally accounting for a minority of the overall change in r^* over the sample. The correlations between each alternative ρ^* measure and the baseline are uniformly very high, indicating that the different measurement approaches identify very similar variation in the pure discounting term over time.

The results in column (3), in particular, provide evidence that our baseline pure discounting

term is not spuriously picking up variation in the expected term premium. We still find the same one-for-one pass-through of $\Delta\rho^*$ to equity yields when starting from this short-term r^* measure, and the full-sample change in ρ^* is nearly identical here as in the baseline.²² Both results are inconsistent with a strong role for term premia in our estimates. The near-identical results for short- and long-term rate measures also have a structural interpretation: [Result 3\(iii\)](#) tells us that the measurement horizon does not matter for permanent shifts in ρ^* , so column (3) suggests that our full-sample estimates of $\Delta\rho^*$ are indeed identifying secular changes. Similarly, columns (4)–(5) suggest that our main full-sample results are not sensitive to the use of VIX² to measure entropy. So while this VIX-based proxy may induce some mismeasurement at higher frequencies, it again appears that this does not affect our results for full-sample secular changes.

In [Appendix Tables B.3 and B.4](#), we show that our results are also robust to measuring Consensus forecasts at *longer* horizons. [Table B.3](#) uses the 10-year horizon. These forecasts are elicited as average expected outcomes over years 6–10, rather than as direct forward-year forecasts. (This is why we use the five-year as our baseline.) The results are nonetheless very close to those in [Table 3](#). [Table B.4](#) measures g^* as the Consensus growth forecast over years 6–10 while keeping r^* at the baseline five-year horizon, allowing expected yields in five years to depend on forward-looking growth at that point. Results are again close to unchanged.

Overall, we find that the main conclusions from [Figure 3](#) are strongly robust to alternative measures used to implement our decomposition.

4.2 Time-Varying Profit Shares

Our main analysis assumes that dividend growth is proportional to output growth. While log dividends and consumption should be cointegrated at a sufficiently long horizon, they do not comove perfectly at all dates or forecast horizons. In recent decades in the U.S., for example, equity cash flows have outpaced GDP and consumption given increases in the corporate profit share of income ([Greenwald, Lettau, and Ludvigson 2025](#)). We briefly consider here how time-varying profit shares affect our analysis, with full results in [Appendix B.4](#).

As discussed in that appendix, even when allowing for an arbitrary dividend-growth process $g_{t+1,d} = d_{t+1} - d_t$ that may be imperfectly correlated with consumption growth, it remains the case that only shocks to the pure discounting term ρ_t^* pass through directly from rates to equity yields. And though the pass-through of growth-rate shocks may be different than in [Result 1](#), it is still the case that such shocks induce weaker pass-through

²²As further illustration that our results hold with shorter-term yields, [Appendix Figure B.4](#) shows r^* constructed from 5-year Consensus forecasts of real bill yields (available over a shorter sample) and compares the resulting ρ^* series to the baseline ρ^* . The two are nearly indistinguishable for the overlapping sample.

than pure-discounting shocks in general. While there may now be pure dividend-growth shocks (i.e., changes to $g_{t,d}^*$ without corresponding changes in g_t^*), these are separate from the interest-rate dynamics considered in our empirical decomposition for r_t^* .

To reconcile this with the importance of profit-share shocks estimated in recent literature, note that such changes may raise contemporaneous cash flows without affecting expected future growth. In this case, prices and cash flows rise together, leaving equity yields unaffected. Alternatively, equity cash-flow growth expectations may have diverged meaningfully from output-growth expectations. While these changes to expected future profit-share growth would be separate from the interest-rate shocks we study, we can nonetheless estimate their magnitude and effects on valuations using two additional U.S. forecast measures: Consensus forecasts on corporate profit growth, and IBES forecasts of long-term earnings growth.

As shown in [Appendix B.4](#), in both cases, profit-share shocks appear to have materialized mainly as changes in current cash flows rather than expected future growth rates, leaving equity yields close to unaffected. Our estimated pass-through of roughly 1/3 of the decline in r_t^* to equity valuations in U.S. data remains unchanged, as do our higher-frequency return regressions when we add profit-share controls. Our main results therefore still apply.

5. Empirical Implications and Economic Mechanisms

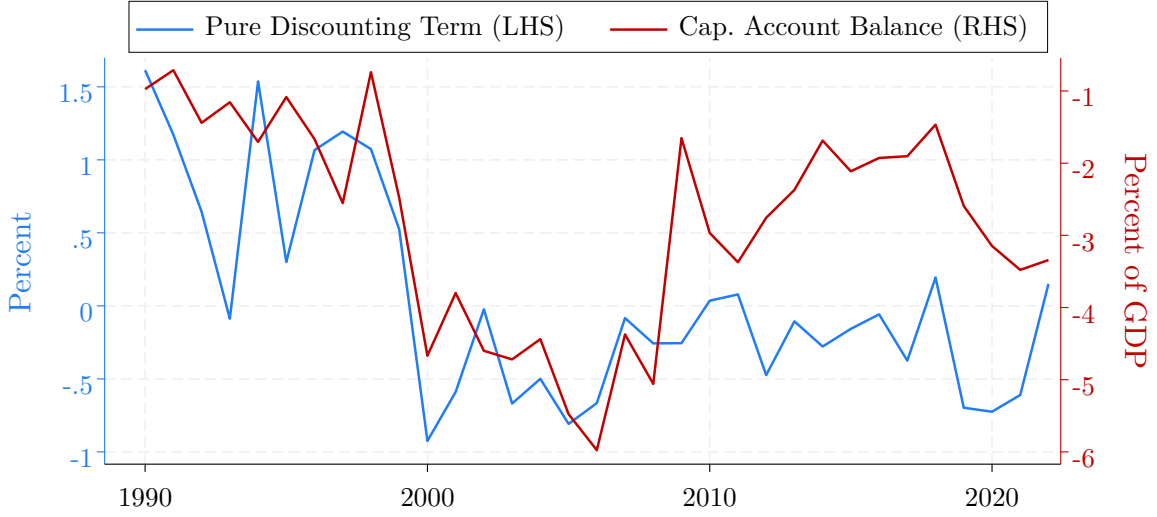
This section considers a series of further applications of our framework and results. In [Section 5.1](#), we examine one potential mechanism underlying variation in the pure discount rate over time: cross-country capital flows. We then use our framework to shed light on three questions that have been debated in recent literature: (1) the “duration-matched” equity premium ([Section 5.2](#)), (2) the effect of decreasing interest rates on the value premium ([Section 5.3](#)), and (3) the potential role of an information effect in explaining stock-price responses to monetary policy news ([Section 5.4](#)).

5.1 Cross-Country Capital Flows

While pure discount-rate shocks align closely with equity valuation changes in our sample, how to interpret these shocks remains an open question. While ρ^* is formally equivalent to the marginal investor’s rate of time preference, shifts in ρ^* are unlikely to reflect such preference changes alone: the notion that Japanese households have become significantly more impatient relative to U.S. households, for instance, seems dubious.²³ As an alternative candidate channel, we consider the potential role of cross-country capital flows in explaining

²³Demographic changes could in principle drive such preference shifts. However, secular-stagnation theories (e.g., [Eggertsson, Mehrotra, and Robbins 2019](#)) often predict that aging should be associated with larger declines in rates. By contrast, we find that $\Delta\rho^*$ in [Figure 3](#) is positively related to a country’s degree of aging.

Figure 6: Pure Discount Rates and Net Capital Flows in the U.S.



Notes: The blue line shows the end-of-year pure discounting term $\hat{\rho}_{t,j}^*$ in the U.S. data, as plotted in Figure 2. The red line shows the U.S. net capital account balance as a share of GDP, measured from the IMF’s “Net Financial Account” series. The figure plots $\hat{\rho}^*$ against annual flows, as Proposition 1 of Caballero, Farhi, and Gourinchas (2008) shows that permanent shocks lead to persistent annual flow effects.

changes in pure discount rates. Such capital flows can in principle affect interest rates in a manner not fully accounted for by domestic fundamentals; since ρ^* absorbs any rate movements not explained by growth or risk, the flows must result in a change in this term.

Concretely, consider a shock that generates strong foreign demand for U.S. assets. For example, following the “global imbalances” model of Caballero, Farhi, and Gourinchas (2008), such demand might arise from foreign investors experiencing a shock to the perceived soundness of their local financial system. The resulting demand for U.S. assets, if relatively inelastic, can push down U.S. interest rates without corresponding changes in U.S. growth expectations or uncertainty. In our decomposition, this would manifest as a decline in the U.S. pure discounting term ρ^* .²⁴ And as long as bond and equity markets are integrated, this change in ρ^* then passes through to equity discount rates and prices following Section 2.

We therefore examine whether capital flows help explain our estimated pure discount-rate changes. We begin with time-series evidence in the U.S. data. As Figure 6 shows, the variation in estimated ρ^* (plotted on the left axis) aligns well with annual net capital flows measured with IMF data (on the right axis, where negative numbers indicate net inflows into U.S. assets). The main decline in the U.S.’s ρ^* occurred in the mid-to-late 1990s and early 2000s, a period coinciding with a sharp decrease in net capital flows.

In Appendix Figure B.5, we analyze whether capital flows help account for pure discount-

²⁴An alternative to the global imbalances view is that pure discount rates and interest rates may have converged toward a common global level given financial integration, with capital flows driving this convergence.

rate changes in the full panel of G7 countries. There is a fairly strong, positive relation between the full-sample change in a country’s ρ^* and the change in its net foreign asset position (calculated from cumulated net capital flows to remove mechanical valuation effects). Countries that received more capital inflows experienced larger declines in ρ^* . Such capital flows thus represent one plausible source of cross-country changes in pure discount rates. That said, capital flows are themselves determined in equilibrium, and they relate to fundamentals in ways that we do not consider here. So while these flows help account for the observed changes in pure discount rates, we leave a deeper analysis of causal drivers for future work.

5.2 Estimating the Equity Premium

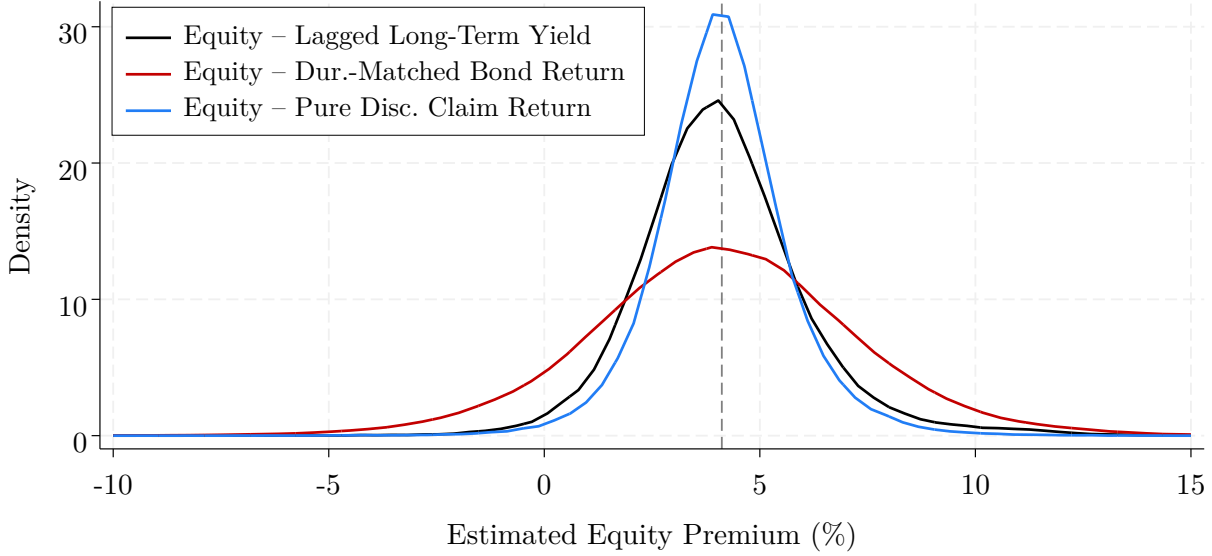
Recent work by [van Binsbergen \(2024\)](#) and [Polk and Vuolteenaho \(2026\)](#) reassesses the performance of the stock market by comparing its realized returns to those of long-term bonds.²⁵ Both papers argue long-term bond returns provide a natural comparison for equities, as equities have a long cash-flow duration themselves. They find that realized stock returns are similar to the realized returns on a duration-matched Treasury portfolio in recent decades, implying a realized excess return on stocks close to 0%. [Van Binsbergen](#) describes this finding as a puzzle, possibly reflecting either low equilibrium compensation for dividend risk or unexpected negative shocks to the path of cash flows.

Our analysis lends support to one aspect of these papers’ argument, as we show in [Result 3](#) that a duration-matched rate measure is indeed the relevant benchmark for equity. That result, however, applies to one specific rate measure: the pure discount rate. Unadjusted bond returns, by contrast, may not be a good benchmark for stocks. Realized stock returns are likely to appear low (or high) when compared to realized returns on long-term bonds: only movements in bond prices (rates) that are driven by pure discounting shocks are fully passed through to stocks. Movements that are driven by growth rates are mostly unrelated to valuations, and movements driven by uncertainty have opposite effects on stocks and bonds. Since bond returns are not incorporated into stock returns in general, stock returns are likely to look high or low relative to bond returns even in relatively long samples.

We formalize this point through simulation studies of an artificial calibrated economy. We consider a generic economy for which the additive decomposition of the log SDF in [\(2\)](#) holds and calibrate it to match the data in terms of volatility of stock prices, growth rates, pure discounting shocks, and risk shocks; see [Appendix B.5](#) for details. We discipline the relation between interest rates, growth, and risk premia based on the empirically observed relations, but we note that using theoretically motivated parameters in the consumption-based version

²⁵[Van Binsbergen](#) considers nominal bonds, while [Polk and Vuolteenaho](#) consider real bonds. In our empirical analysis in [Figure 8](#), we consider only nominal bonds to maximize the available sample.

Figure 7: Simulated Distributions of Average Excess Equity Returns



Notes: This figure shows the results of simulations of calibrated economies. We simulate changes in growth rates, pure discounting terms, and uncertainty in 30 year samples. Based on these realizations of the structural variables, we calculate realized returns on equity and bonds. The figure shows the distributions of average 30-year realized excess returns on stocks across 100,000 simulations. The excess returns are calculated relative to three benchmarks: the lagged long-term interest rate (black), the realized returns on a duration-matched bond portfolio (red), and the realized returns on a duration-matched pure-discounting claim (blue), as defined in equation (23). The true equity premium relative to the long-term yield is shown in the dashed gray line, and the mean of all three distributions is equal to the true premium. Details of the simulations can be found in [Section 5.2](#) and [Appendix B.5](#).

from [Section 2.2](#) gives similar results. Using this model, we run 100,000 simulations of artificial 30-year samples and calculate and compare realized returns on stocks and bonds.

[Figure 7](#) shows that realized stock returns can deviate substantially from realized returns on a duration-matched bond portfolio, and that the difference between the two is a poor estimate of the equity risk premium. The figure shows distributions of average excess returns on stocks relative to three benchmarks. For the first two benchmarks, we consider the ex ante long-term interest rate (black) and the realized return on a duration-matched bond portfolio (red). The two benchmarks deliver the same average excess returns across the simulations (the risk premium of 4%), which is mechanical because the expected return on the duration-matched bond portfolio is the long-run interest rate in our simulations. But the distribution of estimated excess returns is much wider when comparing stock returns to realized bond returns: while observing negative excess returns relative to the ex ante yield is highly unlikely, there is a greater probability (around 10%) of observing negative excess returns when comparing stocks to the realized returns on the duration-matched portfolio. Seen in this light, a near-zero duration-matched equity premium may be somewhat unlikely,

but it is within the range of what should be expected from such exercises.

The results suggest that controlling for duration-matched bond returns is unlikely to improve estimates of the equity risk premium. But our framework suggests that one can, in fact, improve estimates by controlling for changes in the pure discounting term. As in [Results 2–3](#), the effect of a change in the pure discounting term ρ is given by the duration of the stock market times the change in ρ . We can then calculate the realized excess returns net of shocks to the pure discounting term as

$$r_{t,t+n}^{\rho\text{-hedged}} = r_{t,t+n}^{\text{stocks}} - r_{t,t+n}^f - \mathcal{D}(\rho_t - \rho_{t+n}), \quad (23)$$

where $r_{t,t+n}^f$ is the ex ante long-term interest rate. The value $r_{t,t+n}^f + \mathcal{D}(\rho_t - \rho_{t+n})$ can be thought of as the return on a “pure discounting claim,” or a bond that appreciates only when the pure discount rate decreases. The equity return net of this pure discount claim return thus provides an equity return hedged against changes in ρ .

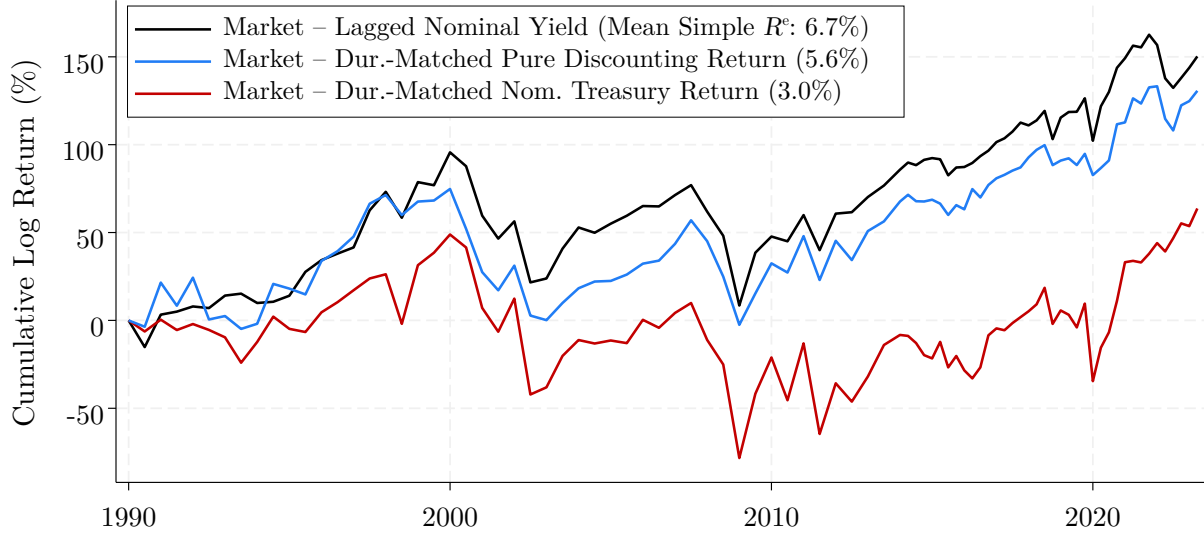
As shown in the third line in [Figure 7](#), realized excess returns net of the pure discounting claim offer a more precise estimate of the equity risk premium than the other approaches. This distribution of excess returns (blue) has the same mean across simulations as the other approaches, so the three methods extract the same equity risk premium on average (the true equity premium). But the ρ -hedged excess returns have a tighter distribution around this true premium: realized stock returns are influenced by unexpected shocks to ρ , and accounting for these shocks brings the realized returns closer to the ex ante equity premium. Because of the market’s long duration, these changes can have a meaningful impact even though the ρ shocks are modest in magnitude. Given our calibrations, the 95% confidence interval for the ρ -hedged excess return is 1.7 to 6.6%, which is tighter than the confidence interval for the standard approach (1.2 to 7.4%). The confidence interval for the duration-matched bond approach is twice as wide as the interval for the ρ -hedged approach (-1.0 to 9.2%).

Motivated by the above results, we estimate the U.S. equity risk premium over our sample period based on the three approaches above: (1) the market return in excess of the long-term nominal bond yield, (2) the market return in excess of the duration-matched nominal Treasury return, analogous to [van Binsbergen \(2024\)](#),²⁶ and (3) the market return hedged against changes in the value of the pure discounting claim.

These cumulative returns are plotted in [Figure 8](#). As can be seen in the black line, the

²⁶Our nominal bond return calculation is somewhat less sophisticated than his. He constructs a bond portfolio with multiple nominal bonds, each weighted in proportion to the value weight of the market’s expected future dividend at the corresponding maturity. By contrast, our counterfactual nominal log bond return is equal to $r_{t+1,n} = y_{t,t+19} - \mathcal{D}(y_{t+1,t+20} - y_{t,t+19})$, where $\mathcal{D} = 19.1$ years from [Table 2](#), and where we take the 19-year zero-coupon yield from [Gürkaynak, Sack, and Wright \(2006\)](#).

Figure 8: Cumulative Excess Returns for the U.S. Market



Notes: This figure shows cumulative log returns on the value-weighted U.S. stock market in excess of three different counterfactual bond returns. The black line shows the return relative to the one-period-lagged 19-year nominal zero-coupon yield from [Gürkaynak, Sack, and Wright \(2006\)](#), where we use a 19-year duration given the results in [Table 2](#). The blue line shows the return relative to the duration-matched pure discounting claim, calculated as in (23) using the estimated $\hat{\rho}_t^*$ from (20). The red line shows the return relative to an unadjusted nominal Treasury security with duration $\mathcal{D} = 19.1$ years; see [footnote 26](#) for details of construction. In all cases, we calculate cumulative log returns for equity and for bonds separately and plot the respective difference. The mean simple excess returns listed in the legend are calculated as average arithmetic (not logarithmic) excess returns for non-overlapping one-year periods.

market has had high average returns relative to the long-term risk-free rate over this period, with a realized annual arithmetic equity premium of 6.7%. The red line shows a version of the finding in [van Binsbergen \(2024\)](#) and [Polk and Vuolteenaho \(2026\)](#): when compared to the holding-period returns on long-term Treasuries, much of this premium disappears. The full-sample average return on this nominal-Treasury-adjusted basis is 3.0%. But before considering the last three years of the sample (which featured increasing interest rates and high equity returns), there was no excess return relative to the duration-matched nominal Treasury: the red line in [Figure 8](#) crosses zero in the fourth quarter of 2020, indicating precisely zero average excess return in the preceding 30 years of the sample.

By contrast, the return on equity in excess of the pure discounting claim, shown in blue, is high and stable. On an annualized basis, this realized excess return is estimated to be 5.6% over this period, only slightly lower than the 6.7% benchmark. As a result, we can conclude that the estimated equity risk premium is close to the historical average even after accounting for the secular decline in the pure discount rate. Over the sample, the pure discount rate goes down by slightly less than 1 pp, which, accounting for the duration effect, leads to a 19% reduction in the realized excess log stock return. This effect has to be averaged out over

the 35-year sample; when converted to simple returns, the estimated equity risk premium decreases by roughly 1% per year.

Taken together, this section argues against comparing realized stock returns to a duration-matched bond counterfactual if the goal is to estimate the equity premium. Subtracting realized bond returns from realized stock returns adds more noise than it subtracts, leading to less precise estimates of the equity premium on average. In our sample, this approach leads to an excess stock return close to 0%. This estimate can appear puzzling, but it is explained by changes in growth rates and risk premia. Once accounting for these changes, realized stock returns are close to the historical equity premium.

5.3 The Value Premium and Interest Rates

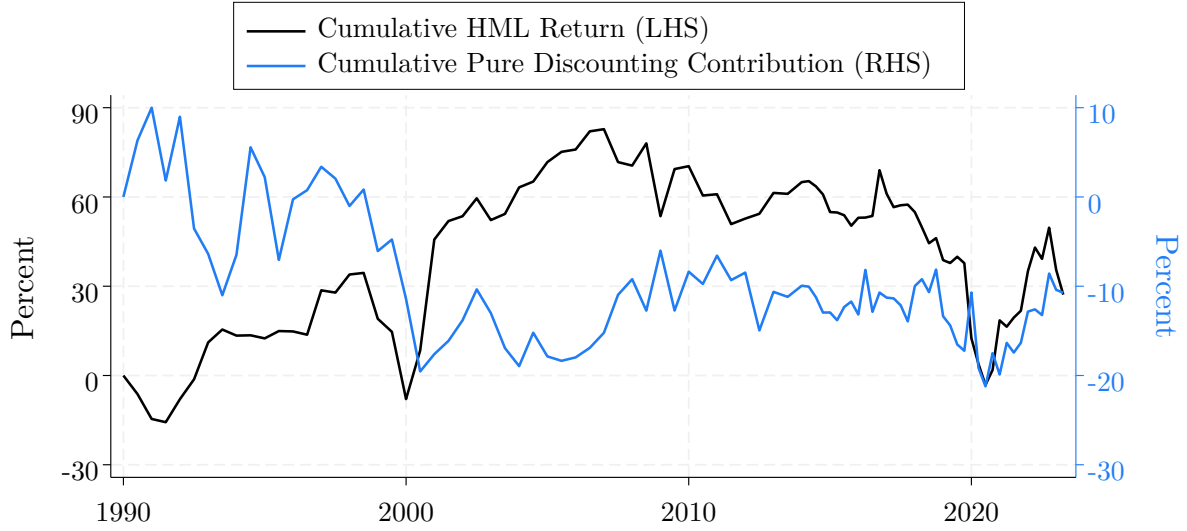
We next turn to a puzzling pattern observed in the cross-section of stocks in recent decades. The value premium — measured as the average return on stocks with high book-to-market ratios minus stocks with low book-to-market ratios, or HML (Fama and French 1993) — has been substantially weaker in recent decades than implied by historical averages. One potential explanation for this underperformance could be that interest rates have dropped, which has led to an unexpected capital gain for the long-duration growth firms, leading growth firms to have performed better than expected *ex ante*.²⁷ On the surface, this effect could be meaningful. Imagine that growth firms have a 30-year longer duration than value firms. A naive calculation would imply that a roughly 3 percentage point drop in interest rates would have led to a 90 percentage point relative outperformance of growth firms. Over a 20-year span, this translates to a relative outperformance of more than 4 percent per year, which is large enough to wipe out effectively the entirety of the historical value premium.

The above calculations are, however, not the full story, as discussed in previous sections. First, while interest rates have dropped by close to 3 percentage points in the U.S., the pure discounting term has dropped by only about 1 percentage point, and it is only this component that should pass through to long-duration assets. Second, we estimate that the spread in duration for value-sorted portfolios is substantially below the 30 years assumed above. The net effect on the realized return on the value factor is therefore substantially smaller.

We illustrate and quantify the effect of changes in the pure discounting term for the value factor in the U.S. data in Figure 9. The figure shows both cumulative returns for the HML factor (in black, corresponding to the left axis), and the estimated contribution of changes in the pure discounting component of real rates (in blue, right axis). This pure-discounting contribution is estimated by regressing HML returns on the change in the residual $\hat{\rho}_{t,j}^*$ from

²⁷This hypothesis is discussed by Maloney and Moskowitz (2021) and Asness (2022), among others.

Figure 9: The Contribution of Pure Discount Changes to Value Factor Returns



Notes: The black line shows the cumulative return on the [Fama and French \(1993\)](#) HML value factor for the U.S. sample since 1990, obtained via Ken French’s website, plotted on the left axis. The blue line shows the contribution we estimate is attributable to the pure discounting component of real rates, plotted in cumulative percent terms on the right axis. For this estimated contribution, we begin with the pure discounting residual $\hat{\rho}_{t,j}^*$, estimated using (19)–(20) following the main specification in column (3) of [Table 1](#), as plotted in [Figure 2](#). We regress three-year HML returns on the three-year change in this residual, along with the three-year change in expected growth rates and the three-year change in the squared VIX (akin to (22)). The estimated contribution from the three-year residual is then the estimated coefficient on $\Delta\hat{\rho}_{t,j}^*$ multiplied by the cumulative change in $\hat{\rho}_{t,j}^*$ since 1990.

equations (19)–(20), controlling for changes in growth rates and uncertainty,²⁸ and then multiplying the estimated coefficient by the cumulative change in $\hat{\rho}_{t,j}^*$ since 1990.

As the figure shows, the effect of the pure-discounting term is modest but non-trivial, reaching a cumulative effect of -20% return at the trough in 2020, but only -10% over the full sample. In addition, the crash and rebound of the value factor from 2020–2023 matches the dramatic changes to the pure discount term experienced over those years at least in timing, if not fully in magnitude: the cumulative HML return over that period is around 30% (on the left axis), while the percent attributable to the pure discounting term is around 10% (on the right axis). So while the pure discounting contribution is often important, it is clearly not the full story explaining the performance of value in recent decades in the U.S. sample.

In [Appendix Figure B.6](#), we exploit our global panel to study what share of the cross-

²⁸This estimation exercise parallels the one in equation (22) for the overall market, with the exception that here we use the change in the residual $\hat{\rho}_{t,j}^*$ estimated in levels from (19) (rather than the residual from the first-difference estimation in (21)). We do so because this allows for straightforward estimation of a cumulative effect of the change in $\hat{\rho}_{t,j}^*$ in levels, as needed for the exercise in [Figure 9](#). Cumulating the effects of three-year changes $\Delta\hat{\rho}_{t,j}^*$, by contrast, would only allow for measurement of the pure discounting contribution every three years (so the blue line in [Figure 9](#) would only show 11 equally spaced points).

country differences in realized value returns since 1990 can be explained by cross-country differences in the evolution of the pure discounting term. The figure shows that value firms in countries that have experienced a larger decrease in the pure discount term have had lower realized premia relative to growth firms over the sample period. The cross-sectional R^2 demonstrates meaningful explanatory power, but it also indicates that the returns to the value factor cannot be fully summarized by changes in the pure discounting term.²⁹ So the pure discounting change is again important for explaining a share, but not the entirety, of the performance of value portfolios across countries.

5.4 Unpacking Monetary Policy Shocks

As a final exercise, we use our decomposition and estimation results to help unpack the effects of surprise changes in short-term interest rates by monetary policymakers. These surprises, when properly measured (e.g., using high-frequency changes in interest rates around policy announcements), are by construction exogenous shocks to short-term nominal rates. But while it may seem intuitive to treat the resulting changes in long-term rates as if they represent pure discounting shocks,³⁰ this is not necessarily a valid assumption: while the change in the short-term rate is exogenous, the long-term yield change depends on changes to the pure discount rate *as well as* changes to perceived long-term growth and uncertainty. That is, the long-term real yield change depends not just on the short-rate shock (and its perceived persistence), but also on the market’s perceived future changes to endogenous outcomes resulting from this shock, as also pointed out by [Chen \(2022\)](#). Further, the sensitivity of stock prices to these shocks does not identify stocks’ cash-flow duration.

A benefit of our framework is that we can directly estimate the perceived effect of any given shock on the separate components of real rates. One approach to this would be to observe the change in expected growth rates around an announcement and then strip out these changes, akin to the approach taken in [Section 3](#). But the timing of the Consensus Economics surveys makes such an approach challenging when considering high-frequency shocks like monetary policy surprises. First, the surveys are conducted infrequently (either every six months or every three months), and forecasters may exhibit inertia in changing their growth-rate forecasts after a given shock. Second, the surveys *prior* to a given shock may be stale by the time of the FOMC meeting, inducing a possibly spurious positive relation between expected-growth revisions and policy surprises; see [Bauer and Swanson \(2023a\)](#).

Instead, we use the fact that we observe three high-frequency asset-price changes on the

²⁹We find that the same results hold — both within and across countries — when considering HML alpha (i.e., on a market-adjusted basis), rather than considering the raw value premium.

³⁰See, for instance, [Kroen et al. \(2022\)](#).

Table 4: Responses of Recovered Rate Components to Monetary Policy Shocks

	(1)	(2)	(3)	(4)
	$\Delta 10\text{y yield}$	$\Delta \text{pure discount } (\widehat{\Delta \rho_t^*})$	$\Delta \text{exp. growth}$	$\Delta \text{VIX}^2 \times 100$
$\text{mps}_t^\perp$	0.42*** (0.06)	0.28*** (0.04)	0.09* (0.05)	1.15*** (0.38)
Obs.	292	292	292	292
R^2	0.37	0.31	0.05	0.04

Notes: This table shows estimates of β from regressions $\Delta x_t = \alpha + \beta \text{mps}_t^\perp + \varepsilon_t$, where Δx_t is the high-frequency change in variable x_t around an FOMC announcement, and $\text{mps}_t^\perp$ is the orthogonalized monetary policy shock from [Bauer and Swanson \(2023b\)](#) in percentage points. In column (1), Δx_t is the 30-minute change in the 10-year nominal yield. In columns (2)–(3), Δx_t are the recovered 30-minute changes in pure discount rates and expected growth based on the high-frequency stock-price and interest-rate changes, as obtained from (24)–(25). In column (4), the outcome variable is the daily change in the squared VIX. Standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. The sample is 1990–2023.

announcement dates: the change in long-term yields $\Delta y_{t,t+n}$, the return on the market r_t^{mkt} , and the change in uncertainty proxied by ΔVIX_t^2 . And our previous estimation provides a mapping from any change in ρ_t , expected growth g_t , and uncertainty VIX_t^2 to a change in long-term yields and stock returns. As a result, this mapping can be inverted to provide an estimate of the change in ρ_t and g_t implied by the observed asset-price changes. For example, a positive market return coinciding with an increase in yields implies that expected growth must have increased by enough, or uncertainty must have decreased by enough, to offset any given increase in the pure discounting term. Given that we can observe the change in uncertainty, these two reactions in fact pin down the required change in both terms.

To implement this idea for the high-frequency data, we start from estimates of the rate decomposition in (21) using three-year changes in the 10-year r^* , and the stock-return regression in (22) using the three resulting terms from the rate decomposition. These give us the loadings mapping changes in growth, uncertainty, and pure discount rates to yield changes and stock returns, and we will assume that these mappings hold as well on policy announcement days.

We then use data from [Bauer and Swanson \(2023b\)](#), who provide changes in 10-year nominal yields, S&P 500 futures returns, and monetary policy shocks (orthogonalized with respect to ex ante predictors) in 30-minute windows around FOMC announcements. Based on the results of [Nakamura and Steinsson \(2018, Table I\)](#), we assume that the change in 10-year nominal yields is equal to the change in 10-year real yields, $\Delta y_{t,t+10}$. Finally, we calculate the daily change in the VIX_t^2 on the announcement day. Using these observed high-frequency changes and our estimated coefficients in the real-rate and stock-return regression, we invert

the following two equations for the two unknowns $\Delta\rho_t$ and Δg_t :

$$\Delta y_{t,t+10} = \Delta\rho_t + \hat{\gamma} \Delta g_t + \hat{\beta}_j \Delta \text{VIX}_t^2, \quad (24)$$

$$r_{t,j}^{\text{mkt}} = \hat{\pi}_\rho \Delta\rho_t + \hat{\pi}_g \Delta g_t + \hat{\pi}_V \Delta \text{VIX}_t^2. \quad (25)$$

We then regress the recovered $\Delta\rho_t$ and Δg_t , as well as $\Delta y_{t,t+10}$ and ΔVIX_t^2 , on the orthogonalized monetary policy shocks $\text{mps}_t^\perp$ from [Bauer and Swanson \(2023b\)](#). [Table 4](#) presents the results. Column (1) shows that a 100-basis-point shock — that is, a shock scaled so that the one-year Eurodollar futures rate changes by +100 bps — results in a 10-year yield increase of 42 bps, nearly identical to [Nakamura and Steinsson \(2018, Table I\)](#). Columns (2)–(4) effectively decompose this change. The majority is attributable to an increase in the pure discount rate of 28 bps. We estimate a weak but nonetheless *positive* change in expected growth of 9 bps. There is also a significant increase in risk.

As a result, we conclude that the average monetary policy shock indeed appears reasonably close to a pure discounting shock, at least in its effect on long-term yields. But there is nonetheless a small, somewhat noisily estimated *positive* expected-growth-rate change estimated as resulting from a contractionary shock. Intuitively, while stock returns decrease following contractionary shocks, they do not decrease on average by quite enough — i.e., they decrease by less than 19% (given an estimated duration of around 19 years) for every one-percentage-point change in long-term yields — to be consistent with a pure discounting shock alone.³¹ As a result, we find some evidence in favor of an information effect on average, using different methods than those used by [Nakamura and Steinsson \(2018\)](#).

The low R^2 in the growth-rate regression indicates that there is meaningful announcement-specific heterogeneity in the perceived effects on growth rates (as well as the other outcome variables): there are some announcements with strong conventional policy responses, and others with strong apparent information effect-type responses. To illustrate this heterogeneity, [Figure B.7](#) in [Appendix B.6](#) shows binned scatter plots of implied changes in the pure discount rate and expected growth rate against monetary policy shocks. The pure discount channel of monetary policy is very clean and consistent: implied ρ_t^* changes increase uniformly in $\text{mps}_t^\perp$. The expected growth channel, meanwhile, appears non-monotonic: large easing shocks generate conventional policy responses (whereby more negative $\text{mps}_t^\perp$ predicts a greater increase in expected growth), while other shocks generate an apparent information effect.

³¹They would in fact need to decrease by more than 19%, given the small positive effect on the VIX.

6. Conclusion

We provide a new framework and measurement tools to decompose any change in real interest rates into mutually exclusive underlying structural changes. According to our decomposition, only pure discounting shocks should pass through perfectly from long rates to equity valuations theoretically. When implemented empirically with long-term survey forecast data and a panel of asset prices, the decomposition works very well: pure discounting shocks pass through one-for-one to equity yields, while the other components of real-rate changes do not.

The recovered pure discounting component of real rates helps us answer a range of questions related to asset pricing, macroeconomics, and secular economic trends observed in recent decades. In the U.S. data, we estimate that a sizable share of the decline in interest rates since 1990 — around 35% — is attributable to the pure discounting term, indicating some meaningful pass-through from declining yields to rising risky-asset valuations. But assuming perfect pass-through, as a range of literature has done, nonetheless overstates the effect of declining interest rates by roughly three times. The partial pass-through we find implies that much of the rise in household wealth (and inequality) was likely non-mechanical.³² Our estimates also suggest that stocks have continued to exhibit a sizable equity premium relative to a duration-adjusted counterfactual. In further analysis, we use our decomposition to speak to higher-frequency equity returns, explain the role of interest rates in the cross-section of stocks, and better understand the perceived effects of monetary policy shocks.

Unpacking the drivers of country-level changes in the pure discounting term in a structural sense, over and above the analysis of capital flows in [Section 5.1](#), will be important for better understanding how to interpret these changes. But in spite of the work to be done on this, our paper provides a clear framework and tools to understand the relationship between stocks and bonds. This bond-stock relationship appears chaotic, both at high frequencies and over the long run, as is apparent from the stock–yield disconnect shown in the left panel of [Figure 1](#). But our simple framework, combined with long-term survey data, works well at isolating a pure discounting component of interest rates that explains both higher-frequency stock returns and longer-term secular changes in equity valuations, as in the right panel of [Figure 1](#).

One implication of our findings is that we can nearly perfectly explain the long-term changes in both interest rates and equities without the need for any additional convenience yield specific to Treasuries. While such market-specific shocks may be quite important for explaining shorter-term fluctuations, “standard” asset pricing evidently works reasonably well at explaining the data at a low frequency.

³²That said, more work needs to be done to understand the pass-through of interest-rate changes to assets other than equity, which are important for many households’ wealth.

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ONLINE APPENDIX:
Interest Rates and Equity Valuations

Niels Joachim Gormsen and Eben Lazarus

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A. Theoretical Derivations and Additional Discussion

This appendix provides proofs, derivations, and discussion of theoretical results as referenced in the main text.

A.1 Additive Log SDF Decomposition

This subsection discusses the additive decomposition for the log SDF in equation (2), and its relation to the decomposition in [Hansen \(2012\)](#). Given our discrete-time environment, for notational simplicity we will in fact more directly build on Proposition 4.2.1 of [Hansen and Sargent \(2022\)](#). That result provides a discrete-time analogue to the continuous-time

decomposition for additive functionals in Theorems 3.1–3.2 of Hansen (2012),³³ which we then apply to our specific setting.

We begin by defining the process $\widetilde{M}$ such that $\widetilde{M}_0 = 1$ and $M_{t+1} = \frac{\widetilde{M}_{t+1}}{\widetilde{M}_t}$, so $m_{t+1} = \widetilde{m}_{t+1} - \widetilde{m}_t$. We consider three settings for fundamental dynamics, mapping to the cases considered in Section 2.2: the first two settings (stationarity with unanticipated breaks, and a drifting steady state) underlie Case I,³⁴ while the third (full stationarity) underlies Case II.

A.1.1 Stationarity with Unanticipated Breaks

Following Hansen and Sargent (2022), begin by defining X_t ($t = 0, 1, 2, \dots$) to be a stationary Markov process of dimension n with transition equation

$$X_{t+1} = \varphi(X_t, W_{t+1}), \quad (\text{A.1})$$

where $\varphi(\cdot, \cdot)$ is a Borel-measurable function and W_{t+1} is a k -dimensional vector of unanticipated shocks satisfying $\mathbb{E}[W_{t+1}] = \mathbb{E}[W_{t+1}|X_t] = 0$. The dynamics in (A.1) induce a transition distribution $\mathcal{P}$ for X .

Given the log SDF $m_{t+1} = \widetilde{m}_{t+1} - \widetilde{m}_t$, assume that the process $\widetilde{m}$ is an additive functional, in the sense that it can be represented as

$$\widetilde{m}_{t+1} = \widetilde{m}_t + \kappa(X_t, W_{t+1}),$$

where $\kappa : \mathbb{R}^n \times \mathbb{R}^k \rightarrow \mathbb{R}$ is a measurable function. Define the unconditional expectation of the increment $\widetilde{m}_{t+1} - \widetilde{m}_t$ to be $\nu = \mathbb{E}[\kappa(X_t, W_{t+1})]$. Define $\bar{\kappa}(x) = \mathbb{E}[\kappa(X_t, W_{t+1})|X_t = x] - \nu$ to be the deviation of the expected increment conditional on $X_t = x$ from its unconditional mean. Using the infinite sum of all such future deviations as of t , define

$$H_t = \underbrace{\kappa(X_{t-1}, W_t) - \nu}_{(\widetilde{m}_t - \widetilde{m}_{t-1}) - \nu} + \sum_{j=0}^{\infty} \mathbb{E}_t[\bar{\kappa}(X_{t+j})], \quad (\text{A.2})$$

and assume that the sum in (A.2) converges in mean square to a finite variable. Next, define

$$h(X_t) = \mathbb{E}_t[H_{t+1}] = \sum_{j=0}^{\infty} \mathbb{E}_t[\bar{\kappa}(X_{t+j})]. \quad (\text{A.3})$$

³³See also Hansen (2019).

³⁴The first setting is a generalization of the first part of Case I, allowing for arbitrary Markov dynamics (rather than the specialized version with conditionally i.i.d. dynamics in Section 2.2) along with unanticipated breaks.

Finally, define the martingale increment

$$\varepsilon_{t+1} = H_{t+1} - h(X_t), \quad (\text{A.4})$$

so that $\mathbb{E}_t[\varepsilon_{t+1}] = 0$ by construction.

Given the above setup and the additional assumption that $\mathbb{E}[\kappa(X_t, W_{t+1})^2] < \infty$, Proposition 4.2.1 of [Hansen and Sargent \(2022\)](#) then gives that $m_{t+1} = \tilde{m}_{t+1} - \tilde{m}_t$ satisfies the following additive decomposition:³⁵

$$m_{t+1} = \nu + h(X_t) - h(X_{t+1}) + \varepsilon_{t+1}. \quad (\text{A.5})$$

The first term represents the linear trend in $\tilde{m}_t$. The second component, $h(X_t) - h(X_{t+1})$, is a stationary difference. The last term is a mean-zero martingale increment.

We now map to our interpretation of the first term as representing discounting and the second term as depending on cash-flow growth. Denote the log cash-flow process by c_t , and assume that

$$\begin{aligned} c_{t+1} - c_t &= \mu_c(X_t) + \sigma_c(X_t)B_c W_{t+1}, \\ X_{t+1} &= A_x X_t + \sigma_x(X_t)B_x W_{t+1}, \end{aligned} \quad (\text{A.6})$$

where $\mu_c(X_t)$, $\sigma_c(X_t)$, and $\sigma_x(X_t)$ are measurable functions. This nests many common specifications for fundamentals. If (i) W_t is a 3×1 i.i.d. standard normal vector, where the first entry contains a shock to contemporaneous cash flows (so only the first entry of the row vector B_c is non-zero), (ii) X_t is a 2×1 vector, where the first entry $X_{1,t}$ affects expected cash-flow growth ($\mu_c(X_t) = \bar{\mu} + X_{1,t}$), the second captures stochastic volatility ($\sigma_c(X_t) = \sigma_x(X_t) = \sqrt{\bar{\sigma}^2 + X_{2,t}}$), A_x is diagonal, and $B_x = (0_{2 \times 1} \tilde{B}_x)$ for a 2×2 diagonal $\tilde{B}_x$, then this represents the long-run risks model of [Bansal and Yaron \(2004\)](#) with stochastic volatility. The [Campbell and Cochrane \(1999\)](#) habit formation model, meanwhile, applies if W_t is scalar i.i.d. standard normal, $\mu_c(X_t) = \bar{\mu}$, $\sigma_c(X_t) = \sigma$, $B_c = 1$, and X_t is the deviation of the log surplus consumption ratio $\tilde{s}_t$ from its long-run mean (with $\sigma_x(X_t)$ being [Campbell and Cochrane](#)'s "sensitivity function" $\lambda(\tilde{s}_t)$). Other settings fit similarly within the framework.

To relate the SDF dynamics to cash flows, assume that the log SDF can be written as

$$m_{t+1} = -\rho - \tilde{\gamma}(c_{t+1} - c_t) + n_{t+1}, \quad (\text{A.7})$$

³⁵Theorem 3.2 of [Hansen \(2012\)](#) gives an exactly analogous decomposition in continuous time, with the added interpretation of h as a finite-second-moment solution to $\lim_{t \searrow 0} \frac{1}{t} \mathbb{E}_0[h(X_t) - h(x)|X_0 = x] = -\bar{\kappa}(x)$, where $\bar{\kappa}(x)$ is the deviation of the local mean of the increment in $\tilde{m}$ from its unconditional mean ν .

for some constant $\tilde{\gamma}$ and process $n_{t+1} = \mu_n(X_t) + \sigma_n(X_t)B_n W_{t+1}$. This is again quite general.³⁶ As in [Section 2.1](#), it holds in a representative-agent, power-utility setting, where $\rho = -\log \beta$ is the time discount rate, $\tilde{\gamma} = \gamma$ is relative risk aversion, and $n_{t+1} = 0$. If the representative agent has Epstein-Zin preferences with elasticity of intertemporal substitution (EIS) ψ , time discount rate ρ , and relative risk aversion γ , [\(A.7\)](#) holds, but now with $\tilde{\gamma} = \frac{1}{\psi}$ and $n_{t+1} = (1/\psi - \gamma)(v_{t+1} - (1 - \gamma)^{-1} \log \mathbb{E}_t[V_{t+1}^{1-\gamma}])$, where V_{t+1} is continuation utility and v_{t+1} is its log.³⁷ In the [Campbell and Cochrane \(1999\)](#) habit setting, [\(A.7\)](#) holds, with $\tilde{\gamma} = \gamma$ again representing risk aversion and $n_{t+1} = -\gamma(\tilde{s}_{t+1} - \tilde{s}_t)$, where $\tilde{s}_t$ is the log surplus consumption ratio. It can also be mapped straightforwardly to various heterogeneous-agent models, such as that of [Constantinides and Duffie \(1996\)](#) or subsequent models.

To relate the above representation to the additive decomposition [\(A.5\)](#), we can construct each of the terms in that decomposition under our assumptions on cash flows in [\(A.6\)](#) and the SDF in [\(A.7\)](#). Define $\nu_c = \mathbb{E}[c_{t+1} - c_t] = \mathbb{E}[\mu_c(X_t)]$ and $\nu_n = \mathbb{E}[n_{t+1}] = \mathbb{E}[\mu_n(X_t)]$, and assume that $\nu_n = 0$. This will hold as long as the additional perturbation n_{t+1} to the log SDF either (i) follows a martingale difference sequence or (ii) features transitory, unconditional-mean-zero disturbances, as is the case in many models.³⁸ The ν in [\(A.5\)](#) is therefore

$$\nu = -\rho - \tilde{\gamma}\nu_c. \quad (\text{A.8})$$

Similarly, split $\bar{\kappa}(X_t)$ into two parts, $\bar{\kappa}(X_t) = \bar{\kappa}_c(X_t) + \bar{\kappa}_n(X_t)$, where $\bar{\kappa}_c(X_t) = -\tilde{\gamma}\mu_c(X_t) + \tilde{\gamma}\nu_c$ and $\bar{\kappa}_n(X_t) = \mu_n(X_t)$. Build up $h_c(X_t)$ and $h_n(X_t)$ accordingly from these $\bar{\kappa}$ functions as in [\(A.3\)](#), and $h(X_t) = h_c(X_t) + h_n(X_t)$. The ε_{t+1} term inherits the remaining martingale-difference components of $-\tilde{\gamma}(c_{t+1} - c_t)$ and n_{t+1} , as in [\(A.4\)](#).³⁹ Define these two processes' respective martingale increment terms as $\varepsilon_{c,t+1}$ and $\varepsilon_{n,t+1}$. Finally, define the martingale difference $\tilde{\varepsilon}_{t+1} = \varepsilon_{t+1} - \varepsilon_{c,t+1} = \varepsilon_{n,t+1}$.

To map the above steps and results to the decomposition in [\(2\)](#), we construct an expanded state vector $\tilde{X}_t = (c_t, X_t)'$, where X_t is the previous state vector. This expanded state vector still follows a Markov process. With non-zero average consumption growth, $\tilde{X}_t$ will no longer be stationary, but its differences will be. This expanded state vector will stand in for the

³⁶It is slightly more general, for example, than the assumption in [Backus, Chernov, and Zin \(2014, eq. \(22\)\)](#).

³⁷In the unit EIS case with $\psi = 1$, $N_{t+1} = \exp(n_{t+1})$ is a martingale, but n_{t+1} is typically not.

³⁸This holds in, for example, the [Campbell and Cochrane \(1999\)](#) model, and see [Borovička, Hansen, and Scheinkman \(2016\)](#) for further discussion. If it does not hold, then the ρ in our decomposition should be understood to contain both the time discount rate and any small component arising from $\mathbb{E}[n_{t+1}]$ (e.g., from a Jensen's inequality correction for the continuation utility term in an Epstein-Zin framework). See [Appendix A.2](#) for a discussion of how this affects the r_t^* decomposition in a tractable alternative case.

³⁹See [Borovička, Hansen, and Scheinkman \(2016\)](#) for explicit characterizations of the martingale components in alternative models.

state vector used in (2). We can now construct our alternative decomposition. Define

$$f(\tilde{X}_{t+1}) - f(\tilde{X}_t) = \tilde{\gamma}(c_{t+1} - c_t) + (h_n(X_{t+1}) - h_n(X_t)). \quad (\text{A.9})$$

Using (A.5), (A.8), and (A.9), we accordingly have our decomposition

$$m_{t+1} = -\rho - (f(\tilde{X}_{t+1}) - f(\tilde{X}_t)) + \tilde{\varepsilon}_{t+1}, \quad (\text{A.10})$$

where $f(\tilde{X}_{t+1}) - f(\tilde{X}_t)$ is a stationary difference and $\tilde{\varepsilon}_{t+1}$ is a mean-zero martingale difference, as in (2). Note that while $\mathbb{E}_t[f(\tilde{X}_{t+1}) - f(\tilde{X}_t)]$ does not depend only on cash-flow growth $c_{t+1} - c_t$ in general, the limiting forward expectation $\lim_{s \rightarrow \infty} \mathbb{E}_t[f(\tilde{X}_{t+s+1}) - f(\tilde{X}_{t+s})]$ used in the trend real-rate decomposition (5) does:

$$\tilde{g}_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[f(\tilde{X}_{t+s+1}) - f(\tilde{X}_{t+s})] = \tilde{\gamma} \mathbb{E}[c_{t+1} - c_t] = \tilde{\gamma} \nu_c. \quad (\text{A.11})$$

Finally, to complete the characterization of this setting's decomposition, we now allow for unanticipated changes in the economic environment. We accordingly denote the transition distribution describing the Markov process X at date t to be $\mathcal{P}_t$, and assume that this distribution governs X_{t+1} and is expected to govern X_{t+1+s} for all $s > 0$. There may then be an unanticipated change in the transition distribution to $\mathcal{P}_{t+1}$, at which point this distribution will govern X_{t+2} and will be expected to govern all future X_{t+2+s} thereafter. Again defining the expanded state vector $\tilde{X}_t = (c_t, X_t)'$, the stationary (under $\mathcal{P}_t$) difference $f(\tilde{X}_{t+1}) - f(\tilde{X}_t)$ in (A.9), and the now potentially time-varying ρ_t in (A.7), our decomposition (A.10) becomes

$$m_{t+1} = -\rho_t - (f(\tilde{X}_{t+1}) - f(\tilde{X}_t)) + \tilde{\varepsilon}_{t+1}. \quad (\text{A.12})$$

This now aligns with (2), with $\tilde{X}_t$ and $\tilde{\varepsilon}_{t+1}$ here in place of X_t and ε_{t+1} in the text. The limiting expectation $\tilde{g}_t^*$ as defined in (A.11) may also be time-varying here, as $\tilde{g}_t^* = \tilde{\gamma}_t \nu_{c,t}$.

A.1.2 Drifting Steady State

Analogous to the drifting-steady-state form of Case I in Section 2.2, we now consider an alternative case in which fundamentals (and resulting expected returns and valuation ratios) follow a random walk, or “drifting steady state.” Define W_{t+1} as in Appendix A.1.1, and assume that the cash-flow process c and one-dimensional process X are modified from (A.6) to now follow

$$c_{t+1} - c_t = \mu_c + X_t + \sigma_c(X_t)B_c W_{t+1},$$

$$X_{t+1} = X_t + \sigma_x(X_t)B_x W_{t+1}.$$

The volatility term $\sigma_x(X_t)$ may be specified so as to ensure that X remains bounded in L^2 in order to rule out explosive dynamics (or one could assume alternative bounded-martingale dynamics for X), but we do not impose this directly.⁴⁰

In place of (A.7), the log SDF follows

$$\begin{aligned} m_{t+1} &= -\rho_t - \tilde{\gamma}(c_{t+1} - c_t) + n_{t+1}, \\ \rho_{t+1} &= \rho_t + B_\rho W_{t+1}, \\ n_{t+1} &= \eta_{t+1} - \eta_t, \end{aligned}$$

where $\mathbb{E}_t[\eta_{t+1}] = \eta_t$. Defining $\tilde{X}_t = (c_t, X_t)'$, we again immediately obtain a decomposition of the form (A.12) and (2):

$$m_{t+1} = -\rho_t - (f(\tilde{X}_{t+1}) - f(\tilde{X}_t)) + \tilde{\varepsilon}_{t+1},$$

where $f(\tilde{X}_{t+1}) - f(\tilde{X}_t) = \tilde{\gamma}(c_{t+1} - c_t) = \tilde{\gamma}(\mu_c + X_t + \sigma_c(X_t)B_c W_{t+1})$ is difference-stationary, and where $\tilde{\varepsilon}_{t+1} = n_{t+1}$ is a martingale difference.

A.1.3 Stationarity

Finally, analogous to Case II in Section 2.2, we consider a setting building on Appendix A.1.1, without unanticipated breaks but with stationary variation in ρ_t . The stationarity of X_t and ρ_t in this setting will imply that infinite-horizon expectations of the discounting and growth terms are constant. As a result, we discuss how to redefine long-run expectations in a manner incorporating persistent time variation in these processes.

We start with exactly the same setting as in Appendix A.1.1, with everything through equation (A.6) unchanged. We modify (A.7) slightly to allow for ρ_t to follow stationary Markov dynamics: assume that ρ_t is an element of the state vector X_t in (A.1) and (A.6), and

$$m_{t+1} = -\rho_t - \tilde{\gamma}(c_{t+1} - c_t) + n_{t+1},$$

with the assumptions on the remaining terms in m_{t+1} unchanged. We also define $\bar{\rho} = \mathbb{E}[\rho_t]$. The above time variation in ρ can be thought of as representing a time-varying subjective discount rate for the representative agent, but it also serves as a stand-in for many other sources of potential variation in the intertemporal marginal rate of substitution. It may arise from demographic changes, a heterogeneous-agents model with time variation in the marginal

⁴⁰Instead, the drifting steady state model is intended as a convenient tool to represent persistent fluctuations in fundamentals, rather than being a reasonable candidate model over an infinite horizon.

investor (and differences in investors' personal discount rates), or as discussed later in the appendix, capital flows from foreign investors.

We can then follow nearly exactly the same steps as in [Appendix A.1.1](#) to obtain the following valid decomposition:⁴¹

$$m_{t+1} = -\rho_t - (f(\tilde{X}_{t+1}) - f(\tilde{X}_t)) + \tilde{\varepsilon}_{t+1},$$

with $f(\tilde{X}_{t+1}) - f(\tilde{X}_t)$ defined as in [\(A.9\)](#) and $\tilde{\varepsilon}_{t+1}$ defined as before [\(A.9\)](#).

In this case, all infinite-horizon expectations defined after the r_t^* decomposition [\(5\)](#) are constant, with $\rho_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[\rho_{t+s}] = \bar{\rho}$, $\tilde{g}_t^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[f(\tilde{X}_{t+s+1}) - f(\tilde{X}_{t+s})] = \tilde{\gamma}\nu_c$, and $L_{t,M}^* = \lim_{s \rightarrow \infty} \mathbb{E}_t[L_{t+s}(M_{t+s+1})] = \sum_{n=2}^{\infty} \frac{\kappa_n(m_{t+1})}{n!}$, where $\kappa_n(m_{t+1})$ is the n^{th} cumulant of the unconditional log SDF distribution. To formalize long-horizon variation in the terms in our r_t^* decomposition in this context, we redefine the above terms as discounted sums:

$$z_t^* = (1 - \delta) \sum_{s=0}^{\infty} \delta^s \mathbb{E}_t[z_{t+s}], \quad (\text{A.13})$$

$$z_t \in \{\rho_t, g_{t+1}, L_t(M_{t+1}), r_{t+1}^f\},$$

where $\delta \in (0, 1)$ is a loglinearization constant and where $g_{t+s+1} \equiv f(\tilde{X}_{t+s+1}) - f(\tilde{X}_{t+s})$. We define the loglinearization constant δ in the appendix for [Section 2.2](#) below, and we discuss how the definition [\(A.13\)](#) maps well to the equity yield decomposition in the stationary case. As $(1 - \delta) \sum_{s=0}^{\infty} \delta^s = 1$, equation [\(A.13\)](#) defines the starred long-run terms as weighted averages of all future expected realizations of z_t , as in [\(13\)](#) in the main text.

A.2 Interest-Rate Decomposition

A.2.1 Benchmark Case

The general version of the decomposition for r_t^* provided in equation [\(5\)](#) in [Section 2.1](#) follows immediately from the SDF decomposition derived and discussed in [Appendix A.1](#), along with equations [\(1\)](#) and [\(3\)](#). For the consumption-based version in [Section 2.1](#), the second expression provided in [\(7\)](#) starts from equation [\(4\)](#) and then applies equation [\(25\)](#) of [Backus, Chernov, and Martin \(2011\)](#), which relates the log SDF's cumulants to the

⁴¹To spell out these steps in further detail: first, with the same definitions and assumptions on ν_c and ν_n as before, the ν in [\(A.5\)](#) becomes $\nu = -\bar{\rho} - \tilde{\gamma}\nu_c$. We now split $\bar{\kappa}(X_t)$ into three parts, $\bar{\kappa}(X_t) = \bar{\kappa}_\rho(X_t) + \bar{\kappa}_c(X_t) + \bar{\kappa}_n(X_t)$, where $\bar{\kappa}_c(X_t)$ and $\bar{\kappa}_n(X_t)$ are as before, and $\bar{\kappa}_\rho = -\mu_\rho(X_t) + \bar{\rho}$. Build up $h_\rho(X_t)$, $h_c(X_t)$, and $h_n(X_t)$ accordingly from these $\bar{\kappa}$ functions as in [\(A.3\)](#), and $h(X_t) = h_\rho(X_t) + h_c(X_t) + h_n(X_t)$. The ε_{t+1} term inherits the remaining martingale components of $c_{t+1} - c_t$ and n_{t+1} as in [\(A.4\)](#), with no additional term for ρ_t given its stationary Markov dynamics, so we define $\tilde{\varepsilon}_{t+1}$ as before. The expanded state vector is again $\tilde{X}_t = (c_t, X_t)'$. The decomposition thus applies as stated.

consumption-growth cumulants in a power-utility setting according to

$$\kappa_{n,t}(m_{t+1}) = (-\gamma)^n \kappa_{n,t}(g_{t+1}). \quad (\text{A.14})$$

See also equation (8) and the preceding equation in [Martin \(2013\)](#), which provides the same interest-rate expression as in (7). Equation (8) then follows directly from the preceding steps. Again see the previous appendix subsection for details on the definitions of the terms in (5) and (8) given each of our sets of assumptions on the dynamics of fundamentals.

A.2.2 Extension with Epstein–Zin Preferences

As discussed above equation (A.8) and in detail in [footnote 38](#), if the additional term n_{t+1} in the log SDF specification (A.7) does not feature $\mathbb{E}[n_{t+1}] = 0$, then the ρ (or ρ_t) in our decomposition should be understood to contain both the time discount rate and any small component arising from $\mathbb{E}[n_{t+1}]$. This does not pose serious issues for either the decompositions or the paper’s interpretation of ρ_t : as discussed in [Section 3.2](#), we do not view shifts in ρ_t in the data as likely to be arising purely from changes in aggregate patience among domestic investors. That said, given the use of Epstein–Zin preferences in [Section 2.2](#), we briefly discuss precisely how such preferences affect the interest-rate decomposition in a tractable case.

In place of (6), we follow [Epstein and Zin \(1989\)](#) and set

$$U_t = \left\{ (1 - \beta_t) C_t^{\frac{1-\gamma}{\theta}} + \beta_t (\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}},$$

where $\theta \equiv (1 - \gamma)/(1 - \frac{1}{\psi})$ and where ψ is the elasticity of intertemporal substitution (EIS). We again set $\rho_t = -\log \beta_t$. In a complete-markets setting in which there is a consumption claim whose value is aggregate wealth, the SDF is

$$M_{t+1} = \left(\beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right)^{\theta} (R_{t+1}^w)^{-(1-\theta)}, \quad (\text{A.15})$$

where R_{t+1}^w is the gross return on the wealth portfolio. If we further assume jointly lognormal and homoskedastic C_{t+1} and R_{t+1}^w , the risk-free rate is then

$$r_{t+1}^f = \rho_t + \frac{1}{\psi} \mathbb{E}_t[g_{t+1}] - \frac{\theta}{2\psi^2} \sigma_g^2 - \frac{1-\theta}{2} \sigma_w^2, \quad (\text{A.16})$$

as in [Campbell \(2018\)](#), equation (6.44), where $\sigma_g^2 = \text{Var}_t(g_{t+1})$ and $\sigma_w^2 = \text{Var}_t(r_{t+1}^w)$. Given

(A.15), the SDF's entropy is

$$L_t(M_{t+1}) = \frac{\theta^2}{2\psi^2}\sigma_g^2 + \frac{(1-\theta)^2}{2}\sigma_w^2 + \frac{\theta(1-\theta)}{\psi}\sigma_{gw}, \quad (\text{A.17})$$

where $\sigma_{gw} = \text{Cov}_t(g_{t+1}, r_{t+1}^w)$. Combining this with (A.16), we can write

$$\begin{aligned} r_{t+1}^f &= \rho_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}) + \frac{\theta(\theta-1)}{2}\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r_{t+1}^w\right) \\ &= \tilde{\rho}_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}), \end{aligned}$$

where $\tilde{\rho}_t = \rho_t + \frac{\theta(\theta-1)}{2}\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r_{t+1}^w\right)$. As a result, (8) still holds, with $\tilde{\rho}_t^*$ in place of ρ_t^* and with $\frac{1}{\psi}$ in place of γ .

The additional term in $\tilde{\rho}_t$ can be thought of as capturing the variance of the innovation to the Epstein–Zin certainty equivalent.⁴² One might thus prefer to group this additional term with the risk component of the SDF instead of ρ_t . To obtain this equivalent representation, note that since $L_t(M_{t+1}) = \frac{1}{2}\text{Var}_t\left(\frac{\theta g_{t+1}}{\psi} - (\theta-1)r_{t+1}^w\right)$ from (A.17), we can write

$$\begin{aligned} r_{t+1}^f &= \rho_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - (1+\omega)L_t(M_{t+1}), \\ \text{where } \omega &\equiv \frac{\theta(1-\theta)\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r_{t+1}^w\right)}{\text{Var}_t\left(\frac{\theta g_{t+1}}{\psi} - (\theta-1)r_{t+1}^w\right)} \end{aligned}$$

is a constant given the homoskedastic setting. Given that our empirical estimation allows for a flexible loading of the risk-free rate on our SDF entropy proxy, this constant of proportionality $1+\omega$ will be incorporated in the empirical versions of the decompositions in such a setting.

As an alternative to the assumption of lognormality after (A.15), assume that g_{t+1} is i.i.d., in which case the log SDF becomes $m_{t+1} = -\rho_t - \gamma g_{t+1}$, exactly as in the benchmark case in the consumption-based version in Section 2.1, so the previous decomposition applies.

A.3 Results for Equity Yields and Duration

Following the main text (and similar to Appendix A.1), we derive our results for equity yields in two settings. We then discuss how the implications for equity duration follow directly, prove the stated results for non-parallel discounting shifts, and discuss our empirical entropy proxy.

⁴²We thank Mike Chernov for helpful discussion related to this point.

A.3.1 Case I (Gordon Growth)

Equity Yields and Risk Premium. Given i.i.d. consumption growth $g_{t+1} = c_{t+1} - c_t$ and $d_{t+1} - d_t = \lambda g_{t+1}$, the setting here mirrors that of [Martin \(2013, Section 1\)](#). He works with a risk premium defined slightly differently than ours: while we use the expected log return and set $rp_t \equiv \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - r_{t+1}^f$, he instead uses the log expected return and considers what we will define as $\tilde{rp}_t \equiv \log \mathbb{E}_t[R_{t+1}^{\text{mkt}}] - r_{t+1}^f$. By definition of entropy, these two risk premia differ by

$$\tilde{rp}_t - rp_t = L_t(R_{t+1}^{\text{mkt}}) = \sum_{n=2}^{\infty} \frac{\lambda^n \kappa_{n,t}(g_{t+1})}{n!}, \quad (\text{A.18})$$

where the second equality uses that $r_{t+1}^{\text{mkt}} = \lambda g_{t+1} + \text{constant}$ (where the constant is in fact ey) in this i.i.d. setting, and then applies the same relation as in [\(A.14\)](#).

Result 1 of [Martin \(2013\)](#), and in particular equation (7), gives that

$$ey_t^* = r_t^* + \tilde{rp}_t^* - \sum_{n=1}^{\infty} \frac{\lambda^n \kappa_{n,t}(g_{t+1})}{n!}, \quad (\text{A.19})$$

where we note that the summation in the last term starts with the first cumulant ($n = 1$) rather than the second as in [\(A.18\)](#). This last term is therefore equal to the cumulant-generating function (CGF) for consumption growth,

$$\mathbf{c}(\vartheta) \equiv \sum_{n=1}^{\infty} \frac{\vartheta^n \kappa_{n,t}(g_{t+1})}{n!}, \quad (\text{A.20})$$

evaluated at $\vartheta = \lambda$. Using [\(A.18\)](#) and [\(A.19\)](#), we have that

$$ey_t^* = r_t^* + rp_t^* - \lambda g_t^*, \quad (\text{A.21})$$

as stated in equation (10) and [Result 1](#).

For the risk-premium expressions in [\(11\)](#), we start from equation (5) of [Martin \(2013\)](#), which gives that $\tilde{rp}_t^* = \mathbf{c}(\lambda) + \mathbf{c}(-\gamma) - \mathbf{c}(\lambda - \gamma)$, and solve for rp_t^* using [\(A.18\)](#):

$$\begin{aligned} rp_t^* &= \tilde{rp}_t^* - (\mathbf{c}(\lambda) - \lambda g_t^*) \\ &= \lambda g_t^* + \mathbf{c}(-\gamma) - \mathbf{c}(\lambda - \gamma) \\ &= \sum_{n=2}^{\infty} \frac{(-\gamma)^n \kappa_{n,t}(g_{t+1})}{n!} - \sum_{n=2}^{\infty} \frac{(\lambda - \gamma)^n \kappa_{n,t}(g_{t+1})}{n!} \\ &= L_t(M_{t+1}) - L_t(M_{t+1} R_{t+1}^{\text{mkt}}). \end{aligned} \quad (\text{A.22})$$

The first line uses the definition of the CGF in (A.18); the second substitutes in the $\tilde{r}p_t^*$ solution above; the third expands the CGFs and uses that the first moments cancel; and the last uses that $m_{t+1} + r_{t+1}^{\text{mkt}} = \text{constant} + (\lambda - \gamma)g_{t+1}$ and applies (A.14). (See also Backus, Chernov, and Martin 2011, p. 2008, for similar steps.) Both lines of (11) follow directly.

To see that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1}R_{t+1}^{\text{mkt}})$ holds in any no-arbitrage setting, one can follow Backus, Boyarchenko, and Chernov (2018, p. 12): take logs of the pricing equation $\mathbb{E}_t[M_{t+1}R_{t+1}^{\text{mkt}}] = 1$ and use the definition of entropy and equation (1) to obtain

$$\begin{aligned} 0 &= \log \mathbb{E}_t[M_{t+1}R_{t+1}^{\text{mkt}}] = \mathbb{E}_t[m_{t+1} + r_{t+1}^{\text{mkt}}] + L_t(M_{t+1}R_{t+1}^{\text{mkt}}) \\ &= \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - r_{t+1}^f - L_t(M_{t+1}) + L_t(M_{t+1}R_{t+1}^{\text{mkt}}). \end{aligned}$$

Rearranging gives that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1}R_{t+1}^{\text{mkt}})$, as stated.

Using (A.21) and (A.22), along with the interest-rate decomposition derived in the previous appendix sections, Result 1 then follows. We note as well that equations (5) and (7) of Martin (2013) also hold with Epstein–Zin utility, so (A.21) and (A.22) are identical in this case, as stated on page 10.

Risk Shocks. In part (iii) of Result 1, it is stated that equity yields change by $-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1$ per unit increase in r_t^* if $\frac{\partial rp_t^*}{\partial L_{t,M}^*}$ is well-defined. The third line of (A.22) shows the need for this qualification: $L_{t,M}^*$ and rp_t^* are both functions of the consumption growth cumulants $\kappa_{n,t}(g_{t+1})$, and there are many potential changes to the different cumulants that generate identical changes in $L_t(M_{t+1})$ but different effects on rp_t^* .⁴³ In certain settings, though, this partial derivative is well-defined. One such setting is when $\gamma = \lambda$: in this case, $rp_t^* = L_{t,M}^*$, so $\frac{\partial rp_t^*}{\partial L_{t,M}^*} = 1$.

Case (i) of the three bond–stock comovement cases described on page 13 is another such setting. This case assumes lognormal growth, so that $\kappa_{n,t}(g_{t+1}) = 0$ for $n > 2$. We therefore have that $rp_t^* = \frac{1}{2}\lambda(2\gamma - \lambda)\text{Var}_t(g_{t+1}) = \frac{\lambda(2\gamma - \lambda)}{\gamma^2}L_{t,M}^*$, so

$$-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1 = -\frac{\lambda(2\gamma - \lambda)}{\gamma^2} + 1 = \frac{(\lambda - \gamma)^2}{\gamma^2},$$

which is strictly positive if $\lambda \neq \gamma$. In a lognormal, power-utility setting, the discount-rate effect of an increase in volatility always dominates, so equity prices increase given an increase in volatility as considered here. But the pass-through of interest rates to equity yields (i.e.,

⁴³This motivates our consideration of the average change using the beta of the risk premium with $L_{t,M}^*$, where we use that $\text{Cov}(rp_t^* - L_{t,M}^*, L_{t,M}^*) = \text{Cov}(rp_t^*, L_{t,M}^*) - \text{Var}(L_{t,M}^*)$. When the partial derivative does not exist, one can take $\frac{\partial rp_t^*}{\partial L_{t,M}^*}$ to represent a stand-in for β_L .

$-\frac{\partial r p_t^*}{\partial L_{t,M}^*} + 1$ above) is nonetheless strictly below one as long as $(\lambda - \gamma)^2 < \gamma^2$, or equivalently $2\gamma > \lambda$, as stated in the text.

We now consider the other two cases introduced on [page 13](#). Case (ii) is based on the rare-disasters model of [Barro \(2006\)](#). Consumption growth is modeled as $g_{t+1} = g_{1,t+1} + g_{2,t+1}$, where the two components $g_{1,t+1}$ and $g_{2,t+1}$ are independent of each other and independent over time. The first component is normal with $\mathbb{E}_t[g_{1,t+1}] = g_1^*$ and variance σ^2 . The second component is a jump component, and we follow [Backus, Chernov, and Martin \(2011\)](#) and assume that it follows a Poisson-normal mixture: a given period has $j \in \mathbb{N}$ jumps with probability $e^{-\omega} \omega^j / j!$ (so ω is the effective jump intensity), and conditional on j , the jump size is normal with mean $-jm$ (with $m > 0$) and variance js^2 . This implies (see p. 2002 of [Backus, Chernov, and Martin](#)) that the consumption-growth CGF is

$$\mathbf{c}(\vartheta) = \vartheta g_1^* + \frac{1}{2} \vartheta^2 \sigma^2 + \omega \left(e^{-\vartheta m + \frac{1}{2} \vartheta^2 s^2} - 1 \right).$$

Given that $L_t(\exp(\vartheta g_{t+1})) = \mathbf{c}(\vartheta) - \vartheta \mathbb{E}_t[g_{t+1}]$ and that in this setting $\mathbb{E}_t[g_{t+1}] = g_1^* - \omega m$, we have that

$$\begin{aligned} L_{t,MR}^* &= L_t(\exp((\lambda - \gamma)g_{t+1})) = \mathbf{c}(\lambda - \gamma) - (\lambda - \gamma)(g_1^* - \omega m) \\ &= (\lambda - \gamma)\omega m + \frac{1}{2}(\lambda - \gamma)^2 \sigma^2 + \omega \left(e^{-(\lambda - \gamma)m + \frac{1}{2}(\lambda - \gamma)^2 s^2} - 1 \right). \end{aligned}$$

Using this,

$$\begin{aligned} ey_t^* &= \rho_t^* + (\gamma - \lambda)g_t^* - L_{t,MR}^* \\ &= \rho_t^* + (\gamma - \lambda)g_1^* - \frac{1}{2}(\lambda - \gamma)^2 \sigma^2 - \omega \left(e^{-(\lambda - \gamma)m + \frac{1}{2}(\lambda - \gamma)^2 s^2} - 1 \right). \end{aligned}$$

Given a change in the average disaster size m , we therefore have

$$\frac{\partial ey_t^*}{\partial m} = (\lambda - \gamma) \omega e^{-(\lambda - \gamma)m + \frac{1}{2}(\lambda - \gamma)^2 s^2}$$

This is strictly positive (so valuations go down given an increase in mean disaster size) iff $\lambda > \gamma$.

Meanwhile, following similar steps for the SDF's entropy and plugging into the risk-free rate decomposition,

$$r_t^* = \rho_t^* + \gamma g_1^* - \frac{1}{2} \gamma^2 \sigma^2 - \omega \left(e^{\gamma m + \frac{1}{2} \gamma^2 s^2} - 1 \right).$$

As a result,

$$\frac{\partial r_t^*}{\partial m} = -\gamma \omega e^{\gamma m + \frac{1}{2}\gamma^2 s^2} < 0,$$

so r_t^* strictly decreases given an increase in mean disaster size. We conclude that such changes induce negative comovement between equity yields and real rates as long as $\gamma < \lambda$.

More generally, for any change in higher ($n \geq 2$) moments $\kappa_{n,t}(g_{t+1})$ for n odd, if $\lambda > \gamma$,

$$\begin{aligned} \frac{\partial L_{t,M}^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{(-\gamma)^n}{n!} < 0, \\ \frac{\partial L_{t,MR}^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{(\lambda - \gamma)^n}{n!} > 0. \end{aligned}$$

Thus, greater negative skewness (i.e., a decrease in $\kappa_{3,t}$) will increase the SDF's entropy $L_{t,M}^*$ but decrease the entropy of the discounted return $L_{t,MR}^*$, and similarly for other higher odd cumulants. Since the risk-free rate decreases in $L_{t,M}^*$ and the equity yield decreases in $L_{t,MR}^*$, these odd-higher-moment shocks will induce negative comovement in the $\gamma < \lambda$ case.

For case (iii) on [page 13](#), with Epstein–Zin utility (and parameters as defined in [Appendix A.2.2](#)), using equations (4)–(5) in [Martin \(2013\)](#) and considering a consumption claim ($\lambda = 1$), we can write

$$\begin{aligned} r_t^* &= \rho_t^* + \sum_{n=1}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} ((1 - 1/\theta)(1 - \gamma)^n - (-\gamma)^n) \\ ey_t^* &= \rho_t^* + \sum_{n=1}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} (-(1 - \gamma)^n/\theta). \end{aligned}$$

So considering any change in higher ($n \geq 2$) moments $\kappa_{n,t}(g_{t+1})$ for n even, we can use the assumptions $\psi > 1$, $\gamma > 1$, and plug in for θ (and use $\theta < 0$ with those assumptions) to get

$$\begin{aligned} \frac{\partial r_t^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{1}{n!} \left[\left(\frac{1}{\psi} - \gamma \right) (1 - \gamma)^{n-1} - (-\gamma)^n \right] < 0, \\ \frac{\partial ey_t^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{1}{n!} (-(1 - \gamma)^n/\theta) > 0, \end{aligned}$$

so equity yields and risk-free rates move in opposite directions, as stated.

Drifting Steady State. We now generalize to the case with arbitrary permanent shocks. If $\mathbb{E}_t[ey_{t+1}] = ey_t \equiv ey_t^*$ (and all underlying fundamentals similarly are martingales), we can follow [Gao and Martin \(2021, eq \(12\)–\(14\)\)](#): since $R_{t+1}^{\text{mkt}} = \frac{D_{t+1} + P_{t+1}}{P_t}$, taking logs and

expectations yields

$$ey_t^* = \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - \lambda \mathbb{E}_t[g_{t+1}] - \log(1 - e^{-ey_t}) + \mathbb{E}_t[\log(1 - e^{-ey_{t+1}})].$$

To a first order for ey_{t+1} around its expectation ey_t , the last two terms cancel, giving equation (12) as stated. Given that r_t^* satisfies the same decomposition (as shown in [Appendix A.1.2](#)), [Result 1](#) therefore holds as stated.

A.3.2 Case II (Stationarity)

In this case, we apply [Result 2](#) of [Gao and Martin \(2021\)](#) directly to obtain that to a first order around the unconditional expectation $\overline{ey} \equiv \mathbb{E}[ey_t]$,

$$ey_t^* \equiv ey_t = (1 - \delta) \sum_{s=0}^{\infty} \delta^s \mathbb{E}_t[r_{t+s+1}^{\text{mkt}} - \lambda g_{t+s+1}], \quad (\text{A.23})$$

where $\delta \equiv e^{-\overline{ey}}$. Therefore, defining rp_t^* (and its underlying components) and g_t^* as in (13), and using the definition for r_t^* and its underlying components from (A.13), the stated decomposition follows. So given the decomposition for r_t^* as shown in [Appendix A.1.3](#), [Result 1](#) again holds as stated.

A.3.3 Equity Duration

The equity duration implications in [Section 2.3](#) for the most part follow from direct application of the previous expressions. For equation (15), we use that $\mathbb{E}_t[D_{t+s}] = D_t e^{s\lambda g^*}$ and then evaluate the resulting series. The remaining derivatives in (16) and [Result 2](#) then follow directly from (14)–(15).

A.3.4 Non-Parallel Discounting Shifts

We now prove [Result 3](#) and additional results described in [Section 2.4](#). As in that subsection, we start at a flat steady state (as would arise in the conditionally i.i.d. setting in Case I) for pure discount rates, growth, and risk, and we denote $\rho^*(s) \equiv \mathbb{E}_t[\rho_{t+s-1}]$. Define $\bar{\rho}^*(s) \equiv \frac{1}{s} \sum_{u=1}^s \rho^*(u)$ for the average one-period pure discount rate out to horizon s , and again define $\delta \equiv e^{-ey^*}$, which is equivalent to the loglinearization constant used in (A.23). Given this definition, $\mathcal{D} = 1/(1 - \delta)$ and $\delta = 1 - 1/\mathcal{D}$ by (15). We also make repeated use of the geometric sums $\sum_{s \geq 1} \delta^{s-1} = \frac{1}{1-\delta}$, $\sum_{s \geq 1} s \delta^{s-1} = \frac{1}{(1-\delta)^2}$, and $\sum_{s \geq 1} s^2 \delta^{s-1} = \frac{1+\delta}{(1-\delta)^3}$.

Compounding the one-period log SDF, and taking future pure discount rates for now at

their conditional means $\rho^*(u)$,⁴⁴

$$M_{t,t+s} = \exp\left(\sum_{u=1}^s -(\rho^*(u) + \gamma g_{t+u})\right).$$

Further, the price of the s -period dividend strip is

$$P_s \equiv \mathbb{E}_t[M_{t,t+s}D_{t+s}] = D_t e^{-\sum_{u=1}^s \rho^*(u)} \mathbb{E}_t\left[\exp\left((\lambda - \gamma) \sum_{u=1}^s g_{t+u}\right)\right] = D_t e^{-s\bar{\rho}^*(s) + s\mathbf{c}(\lambda - \gamma)},$$

where $\mathbf{c}(\cdot)$ is the CGF defined in (A.20), and where the last equality uses that at the flat steady state, the growth and risk contribution is the same at every horizon. The s -period equity yield, defined by $P_s = D_t e^{-s ey^*(s)}$, is then $ey^*(s) = \bar{\rho}^*(s) - \mathbf{c}(\lambda - \gamma)$. Similarly, for the s -period real bond, $\mathbb{E}_t[M_{t,t+s}] = e^{-s\bar{\rho}^*(s) + s\mathbf{c}(-\gamma)}$, so its yield is $r^*(s) = \bar{\rho}^*(s) - \mathbf{c}(-\gamma)$. Substituting $\mathbf{c}(-\gamma) = -\gamma g^* + L_M^*$ from (7), and $\mathbf{c}(\lambda - \gamma) = (\lambda - \gamma)g^* + L_{MR}^*$ from (A.22), then gives

$$\begin{aligned} r^*(s) &= \bar{\rho}^*(s) + \gamma g^* - L_M^*, \\ ey^*(s) &= \bar{\rho}^*(s) + (\gamma - \lambda)g^* - L_{MR}^*. \end{aligned} \tag{A.24}$$

This is the maturity-specific version of Result 1. Holding growth and risk fixed, a change $d\bar{\rho}^*(s)$ in the average forward discount rate out to horizon s passes one-for-one into both the s -period real bond yield and the s -period strip yield, at every horizon.

At the flat steady state, $\bar{\rho}^*(s) = \rho^*$ and $ey^*(s) = ey^*$ for all s , so $P_s/D_t = \delta^s$ and $P/D_t = \sum_s \delta^s = \frac{1}{e^{ey^*} - 1}$, as in (14). The strip value weights are then

$$w_s \equiv \frac{P_s}{P} = (e^{ey^*} - 1) e^{-s ey^*} = (1 - \delta) \delta^{s-1},$$

as stated in the text. These weights generate a mean strip-weighted horizon of $\sum_s s w_s = \mathcal{D}$, consistent with (15) and as stated in the text, and the strip-weighted horizon variance is $\text{Var}_w(s) = \sum_s s^2 w_s - \mathcal{D}^2 = \frac{\delta}{(1-\delta)^2} = \mathcal{D}(\mathcal{D} - 1)$. We can now prove the three stated parts of

⁴⁴We do not require $\rho_{t+u-1} = \rho^*(u)$ ex post, but the current proof conducts a perturbation for which this is without loss of generality. Formally, for a deterministic sequence ζ_u , we define a perturbed economy in which the one-period pure discount rate applying to the period ending at $t+u$ is shifted in every state by $\varepsilon \zeta_u$, while the conditional distribution of growth and all other SDF terms is unchanged. Taking the Gâteaux derivative at $\varepsilon = 0$ with $\zeta_u = d\rho^*(u)$, the perturbed strip price at horizon s' satisfies $P_{s'}(\varepsilon) = P_{s'}(0) \exp(-\varepsilon \sum_{u \leq s'} \zeta_u)$. Thus $d \log P = -\sum_u W_u \zeta_u$, where $W_u \equiv \sum_{s' \geq u} P_{s'}/P$ is the share of total equity value for cash flows at maturities u and beyond. Since the increment ζ_u is deterministic, this holds regardless of the distribution of ρ_{t+u-1} or its correlation with growth. In other words, setting pure discount rates at their conditional means in the SDF affects the baseline SDF but not the response of strip prices or equity.

the result in turn.

General Shift. Consider a shift $\{d\rho^*(s)\}$ of the forward pure discounting curve. Writing $\log P = \log D_t + \log \sum_{s \geq 1} \exp(-s\bar{\rho}^*(s) + s\mathbf{c}(\lambda - \gamma))$, the forward rate $\rho^*(s)$ enters the exponent of $P_{s'}$ with coefficient -1 exactly when $s' \geq s$, so

$$\frac{\partial \log P}{\partial \rho^*(s)} = -\frac{\sum_{s' \geq s} P_{s'}}{P} \equiv -W_s,$$

where W_s is the share of market value for cash flows at horizons s and beyond. To first order, the shift moves the price by $d \log P = -\sum_{s \geq 1} W_s d\rho^*(s)$.⁴⁵ Summing by parts,

$$\sum_{s \geq 1} W_s = \frac{1}{P} \sum_{s' \geq 1} s' P_{s'} = \sum_{s'} s' w_{s'} = \mathcal{D},$$

which is equal to the value-weighted mean maturity. Further, at the flat baseline, we have that $W_s = \sum_{s' \geq s} (1 - \delta)\delta^{s'-1} = \delta^{s-1}$, so $W_s/\mathcal{D} = (1 - \delta)\delta^{s-1} = w_s$ coincides with the strip value weight itself. The price response to an arbitrary shift is therefore

$$d \log P = -\mathcal{D} \sum_s w_s d\rho^*(s).$$

The equity-yield response follows from (9). Since $ey = \log(P + D_t) - \log P$ and D_t is unchanged by a shift in the discounting curve, $d ey = -\frac{D_t}{P+D_t} d \log P = -\frac{1}{\mathcal{D}} d \log P$, using $\frac{D_t}{P+D_t} = 1 - e^{-ey^*} = 1/\mathcal{D}$ at the steady state. Combining the two,

$$d ey^* = \sum_s w_s d\rho^*(s), \quad d \log P = -\mathcal{D} d ey^*,$$

which give (17) as stated. This proves part (i).

Affine Shift and the Relevant Horizon. For an affine shift $d\rho^*(s) = a + bs$, the horizon-weighted average shift is $\sum_s w_s d\rho^*(s) = a + b \sum_s s w_s = a + b\mathcal{D} = d\rho^*(\mathcal{D})$, equal to the shift evaluated at the (potentially non-integer) horizon $s = \mathcal{D}$. So $d ey^* = d\rho^*(\mathcal{D})$ and $d \log P = -\mathcal{D} d\rho^*(\mathcal{D})$, which together give (18) as stated, and the relevant horizon is equity duration. This proves part (ii).

To see how the affine shift first-order approximates a general shift (as in footnote 11), we

⁴⁵This is the directional derivative in the direction of the shift. We assume that the derivative is well defined (as would be the case, for instance, for a shift of at most polynomial growth in s , in which case W_s decays geometrically). We read the affine shifts considered below as this derivative, or equivalently as a shift that is affine out to a horizon $H \gg \mathcal{D}$ and bounded beyond.

can Taylor expand $d\rho^*(s)$ around the mean horizon $s = \mathcal{D}$:

$$d\rho^*(s) = d\rho^*(\mathcal{D}) + d\rho^{*'}(\mathcal{D})(s - \mathcal{D}) + \frac{1}{2} d\rho^{*''}(\mathcal{D})(s - \mathcal{D})^2 + \dots$$

Taking the strip-weighted average and using $\sum_s w_s = 1$, $\sum_s w_s(s - \mathcal{D}) = 0$ (the weights have mean $\mathcal{D}$), and $\sum_s w_s(s - \mathcal{D})^2 = \text{Var}_w(s) = \mathcal{D}(\mathcal{D} - 1)$,

$$\sum_s w_s d\rho^*(s) = d\rho^*(\mathcal{D}) + \frac{1}{2} d\rho^{*''}(\mathcal{D}) \mathcal{D}(\mathcal{D} - 1) + \dots$$

The linear term is equal to zero in general, because the weights again have mean $\mathcal{D}$. An affine shift ($d\rho^{*''} = 0$) then gives $\sum_s w_s d\rho^*(s) = d\rho^*(\mathcal{D})$ exactly, while a general shift has second- and higher-order terms.

Parallel Shift. If $d\rho^*(s) = d\rho^*$ for all s , then $\sum_s w_s d\rho^* = d\rho^*$. Thus $d\text{ey}^* = d\rho^*$ and $d\log P = -\mathcal{D} d\rho^*$, and $d\bar{\rho}^*(s) = d\rho^*$ at every horizon by (A.24). Every horizon is then equally relevant, and the response recovers [Result 1\(i\)](#) and [Result 2\(i\)](#). This proves part (iii).

Other Rate Measures. The strip value weights also allow us to characterize the response of equity to changes in other (potentially composite) measures of pure discount rates, including those discussed in the text. Since $W_1 = 1$, a shift in the one-period pure discount rate alone moves $\log P$ by its own size rather than by $\mathcal{D}$ times it, so the pass-through to the equity yield is $w_1 = 1/\mathcal{D}$. A shift in horizons $s \leq k$ has total weight $1 - \delta^k$. For the multi-period forwards we use in the data, the forward rate averaged over periods m to n , $\bar{\rho}^*(m, n) \equiv \frac{1}{n-m+1} \sum_{s=m}^n \rho^*(s)$, moves under an affine shift by $a + b\frac{m+n}{2}$, which equals $a + b\mathcal{D} = d\rho^*(\mathcal{D})$ if and only if its midpoint satisfies $\frac{m+n}{2} = \mathcal{D}$, as stated in [Section 2.4](#). More generally, any measure $dM = \sum_s q_s d\rho^*(s)$ with $\sum_s q_s = 1$ reproduces the response under affine shifts if and only if its mean horizon $\sum_s q_s s$ equals $\mathcal{D}$.

A.3.5 Empirical Entropy Proxy

As stated in [Section 3](#) (see [page 19](#)), if the market is growth-optimal and the distribution of log growth is symmetric, then $L_{t,M,j}^* = L_{t,R,j}^*$. To see this, note from (A.14) and (A.18) that

$$\begin{aligned} \kappa_{n,t}(m_{t+1}) &= (-\gamma)^n \kappa_{n,t}(g_{t+1}), \\ \kappa_{n,t}(r_{t+1}^{\text{mkt}}) &= \lambda^n \kappa_{n,t}(g_{t+1}). \end{aligned}$$

Growth optimality requires $\lambda = \gamma$, and a symmetric log growth distribution implies that $\kappa_{n,t}(g_{t+1}) = 0$ for all odd n (aside from $n = 1$, which does not enter into the entropy sum).

And for even n , given $\gamma = \lambda$, we have $(-\gamma)^n = \lambda^n$. As a result, $\kappa_{n,t}(m_{t+1}) = \kappa_{n,t}(r_{t+1}^{\text{mkt}})$ for all n , and so $L_{t,M,j}^* = L_{t,R,j}^*$, as stated. This (along with [Martin 2017](#), Result 3) motivates our use of the squared VIX to proxy for the SDF entropy term $L_{t,M}^*$ in estimating our r_t^* decomposition.⁴⁶

B. Additional Empirical Details and Results

This appendix provides additional measurement details and empirical results as referenced in the main text.

B.1 Baseline Measurement Details

[Section 3.1](#) explains most of our data sources and variable definitions for the baseline analysis. Here, we provide additional details on two aspects of the data mentioned in the text.

VIX Measurement. The squared VIX is defined for horizon $T - t$ as

$$\text{VIX}_{t,T}^2 = \frac{2R_{t,T}^f}{T-t} \left(\int_0^{F_{t,T}} \frac{\text{put}_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{\infty} \frac{\text{call}_{t,T}(K)}{K^2} dK \right),$$

where $F_{t,T}$ is the forward price and $\text{put}_{t,T}(K)$ and $\text{call}_{t,T}(K)$ are prices of European put and call options with strike K expiring at T . To implement this formula, we use a cleaned version of a global panel of index option prices from OptionMetrics. As in the main text, the sample, data filters, and implementation approach are taken from [Gandhi, Gormsen, and Lazarus \(2025\)](#); see that paper for full details.

Our options data are available starting in 1990 in the U.S. sample, but the samples for other countries start between 2002 and 2006. To obtain a full sample corresponding to the forecast data, we project $\text{VIX}_{t,j}^2$ in the available sample onto realized volatility in the country j index return, and then obtain predicted values $\widehat{\text{VIX}}_{t,j}^2$ using the observed volatility for any dates in the sample for which we cannot calculate VIX directly.

Equity Yields and Data Availability. As in the text, the equity yield $ey_{t,j}$ is measured by starting with the five-year earnings-to-price ratio $\bar{E}_{t-4,t,j}/P_{t,j} = [(E_{t-4,j} + \dots + E_{t,j})/5]/P_{t,j}$, where earnings and prices are calculated on a value-weighted basis for all available traded stocks in the country. Earnings $E_{t,j}$ are defined as net income for the full calendar year corresponding to date t , while prices $P_{t,j}$ are end-of-period aggregate market capitalizations.

⁴⁶[Martin](#)'s result is for risk-neutral entropy, so this proxy additionally requires an assumption of a constant entropy risk premium (or, in the log-normal case, variance risk premium) over the sample. This is likely a reasonable approximation for the full-sample differences considered in the secular trends analysis, though it likely induces further measurement error for the higher-frequency analyses.

We then scale this $\bar{E}_{t-4,t,j}/P_{t,j}$ by 0.5, very close to the unconditional average payout ratio of 0.494 in our post-1990 sample, to obtain our final measure of equity yields $ey_{t,j}$. Algebraically, $ey \equiv \log(1 + D/P) \approx D/P = (D/E) \times (E/P)$. Our unconditional average payout ratio is calculated as the average ratio of five-year-average common dividends to five-year-average net income, to put the dividend and earnings figures in common terms, and the average is across all years and G7 countries starting in 1990.

In some countries, the share of publicly traded companies with available earnings data is low in the early part of our sample. We drop any country-year equity yield observations with such coverage issues. The resulting samples start in 1990 for the U.S. and Canada; 1992 for the U.K.; 1993 for Japan; 1994 for France and Germany; and 1998 for Italy. Any estimated relationship between changes in earnings yields and changes in interest rates uses a country-specific start date consistent with the beginning of this equity yield sample; for example, the r^* difference for Italy in [Figure 3](#) is the difference starting from 1998, consistent with the difference calculated for its equity yields. While all our analyses use data through 2023, the need for full-year net income data to calculate $ey_{t,j}$, along with the fact that our data is only available through part of 2023, means that we measure equity yields through the end of 2022. In all cases where we calculate full-sample differences in the real rate (and its components), we again match this full-sample difference with the actual available sample for our equity data. (Any analysis focusing solely on the real-rate data — for example, the first-stage regression in [Table 1](#) — uses all the data through 2023.)

B.2 Details on Block Bootstrap Inference

For the top panel of [Table 3](#) in [Section 4.1](#), we compute standard errors and confidence intervals using a block bootstrap that clusters at the year level. We draw years with replacement from the original sample, and for each bootstrap replication, we then construct a resampled panel by including all country-quarter observations from each drawn year. We then run the first-stage regression (19) using the specification in column (3) of [Table 1](#) and obtain the estimated pure discounting observations as in (20). Given these $\hat{\rho}_{t,j}^*$ values and the resampled equity yields, we then collapse to the first and last available observation by country to calculate full-sample differences and run the regression in the top panel of [Table 3](#).

This last step is conducted in a manner that accounts for the pronounced trends in our sample. In particular, before collapsing to the first and last observation by country to calculate full-sample differences, we sort the resampled data so that the temporal ordering of the years is preserved from the original sample. Specifically, if the bootstrap draw selects years $\{t_1, t_2, \dots, t_T\}$ from the original sample (where $t_1 \leq t_2 \leq \dots \leq t_T$), we label these as $\{1990, 1991, \dots, 1990 + T - 1\}$ in the resampled data (where $T = 34$). This means that early

observations in the bootstrap sample are drawn from years that appeared early in the original sample, and late observations are drawn from years that appeared late in the original sample. So when collapsing the data to the first observation in the year labeled 1990 and the last observation in the year labeled $1990 + T - 1$, we preserve the nonstationarity of the original data (i.e., the pronounced change from beginning to end), while nonetheless allowing the possibility of moderate (local) changes in the start and end date.⁴⁷

This approach implements a form of the *local block bootstrap* proposed and analyzed by Paparoditis and Politis (2002) and Dowla, Paparoditis, and Politis (2013) for inference with non-stationary time series. The classic block bootstrap assumes stationarity, which is violated in our setting.⁴⁸ The local block bootstrap instead ensures that resampled observations for each time period are drawn from a local window around that period in the original sample, preserving any local dependence structure and accommodating slowly varying distributions.

We use 10,000 bootstrap replications for all reported standard errors and significance tests. Statistical significance is determined by inverting basic bootstrap confidence intervals, computed as $(2\hat{\beta} - \beta_{1-\alpha/2}^*, 2\hat{\beta} - \beta_{\alpha/2}^*)$, where $\hat{\beta}$ is the full-sample point estimate and β_x^* denotes the x percentile of the bootstrap distribution.

B.3 Robustness Measures

This section describes the alternative input data used in Section 4 in more detail:

- **Survey of Professional Forecasters (SPF):** We obtain mean responses for three SPF forecast variables via the Federal Reserve Bank of Philadelphia’s website: the average annual 10-year nominal Treasury yield over the next 10 years, average CPI inflation over the next 10 years, and average real GDP growth over the next 10 years. These forecasts are provided once per year in the first quarter of each year. We calculate the forecasted average real yield by subtracting the CPI forecast from the Treasury yield forecast, and we use this as our alternative measure of r^* . We use the GDP forecast as our alternative measure of g^* . These forecasts are only available starting in 1992; to align the available sample with our main analysis, we run regressions of the SPF variables on the corresponding variable from the baseline Consensus data and

⁴⁷For example, if the first year in the resampled data set is 1992, then the full-sample change will be calculated starting from 1992. In our setting, 99.8% of replications begin and end within the first and last five years of the original sample, respectively. The order-preserving feature of this bootstrap helps explain the strong statistical significance in the main regression despite the small apparent sample size of 7 countries: we in fact have many observations per country with which to estimate the pure discount rate, and its behavior from the start to the end of the sample is quite robust.

⁴⁸This violation does not affect inference in cases where the ordering of the data does not matter. So for the bootstrapped standard errors reported in Section 3 (for which the regressions are invariant to the data ordering), we use this standard block bootstrap.

calculate the fitted values, which we use for the 1990 and 1991 observations.

- **Del Negro et al. (2019):** For this alternative short-term r^* measure, we use the baseline cross-country natural-rate estimates provided by Del Negro et al. (2019). These are model-based estimates of the trend short-term real rate, provided as an annual panel that runs through 2019. To extend the U.S. series to the end of our sample to calculate the full-sample change in ρ^* , we project the Del Negro et al.-based ρ^* onto the baseline ρ^* , then use the fitted value for the post-2019 observations.
- **GARCH(1,1):** For each country, we fit a GARCH(1,1) model to their daily equity returns data, and we use this to forecast variance at the six-month horizon. We then use this in place of VIX^2 in the interest-rate decomposition. Results are also quantitatively nearly identical when using the GARCH variance forecast *in addition to* the VIX in the decomposition, rather than as a replacement.
- **Baker, Bloom, and Davis (2016) Uncertainty Index:** We obtain the country-level economic policy uncertainty series from <https://www.policyuncertainty.com> for the set of available countries in our sample, and we use these data in place of VIX^2 in the interest-rate decomposition. Results are again quantitatively nearly identical when using the uncertainty index *in addition to* the VIX in the decomposition, rather than as a replacement.

B.4 Robustness to Time Variation in Profit Shares

This section presents and discusses the robustness results introduced in Section 4.2, first theoretically and then empirically. For the theoretical results, we first note that our interest-rate decomposition in Result 1, $r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*$, is unchanged, as aggregate growth g_{t+1} is the relevant outcome for the SDF. For equity cash flows, denote dividend growth by $g_{t+1,d} = d_{t+1} - d_t$. Rather than imposing $g_{t+1,d} = \lambda g_{t+1}$, we now allow for an arbitrary dividend growth process. So it may be the case that $\text{Corr}(g_t^*, g_{t,d}^*) < 1$. Our equity-yield decomposition now becomes

$$\begin{aligned} ey_t^* &= \rho_t^* + \gamma g_t^* - g_{t,d}^* + (rp_t^* - L_{t,M}^*) \\ &= \rho_t^* + \gamma g_t^* - g_{t,d}^* - L_{t,MR}^*. \end{aligned}$$

As before, only shocks to the pure discounting term ρ_t^* pass through directly from rates to equity yields. And though the pass-through of growth-rate shocks may be different than in Result 1, it is still the case that such shocks induce weaker pass-through than pure-discounting

shocks as long as $\text{Corr}(g_t^*, g_{t,d}^*) > 0$. While there may now be pure dividend-growth shocks (i.e., changes to $g_{t,d}^*$ without corresponding changes in g_t^*), these are entirely separate from the interest-rate dynamics considered in our empirical decomposition for r_t^* .

For estimating the effect of profit-share changes on valuations empirically, we use two sources of additional forecast data. First, the long-term Consensus Economics forecast data provides nominal corporate profit growth forecasts since 1998 for only the U.S. sample. Using this, we construct a proxy for real $g_{t,d}^*$ by subtracting the inflation forecast from the nominal profit-growth forecast for year $t + 5$. In this available post-1998 U.S. sample,⁴⁹ profit-growth forecasts have in fact fallen by meaningfully more than output-growth forecasts:

$$\Delta g_t^* = -0.50, \quad \Delta g_{t,d}^* = -1.26.$$

With a leverage parameter of $\lambda \approx 2$ — close to the estimated coefficient on expected growth in the real-rate decomposition in [Table 1](#), consistent with the insignificant equity effect estimated in [Table 2](#) — we accordingly estimate that the change in the expected profit share $\pi_t^* \equiv g_{t,d}^* - \lambda g_t^*$ has been very close to 0 over this sample. As a result, our main conclusions for the U.S. data are unchanged: profit-share shocks have affected current cash flows but not expected future growth rates, leaving equity yields close to unaffected. As a result, the previous estimated effect of the pure discounting residual, and the resulting pass-through of roughly 1/3 of the decline in r_t^* to equity valuations in U.S. data, remains valid.

As a secondary check on this analysis, we obtain aggregate long-term earnings growth (LTG) forecasts for U.S. equities from [Nagel and Xu \(2022\)](#). These are analyst forecasts for earnings growth for U.S. equities, which [Nagel and Xu](#) aggregate to the index level and convert to real terms by subtracting forecasted inflation. Using this series as our second proxy for $g_{t,d}^*$, for the full sample over which we can implement our real-rate decomposition and measure equity yields, we estimate that $\Delta g_t^* = -0.70$, $\Delta g_{t,d}^* = -0.60$.⁵⁰ If one again assumes a leverage parameter of about $\lambda = 2.5$, this implies that there has been a modest increase in the profit-share term $\Delta \pi_t^*$, but our main results are largely unaffected.

⁴⁹This post-1998 sample corresponds to a period during which the majority of the decline in expected output growth took place in the U.S. forecasts.

⁵⁰We prefer the Consensus Economics data because the LTG-based forecasts are much more volatile than the other forecast series (see [Bordalo et al. 2024](#)), so the full-sample differences are highly sensitive to the start and end date. Starting the sample in 1992 instead of 1990 gives $\Delta g_{t,d}^* = -1.88$; meanwhile, ending the sample a year later gives $\Delta g_{t,d}^* = 0.14$. Given such large cyclical LTG forecast variation ($\sigma_{\text{LTG}} = \text{SD}(\text{LTG}) = 2.0$, vs. $\sigma_{\text{profit}} = 0.8$, $\sigma_{\text{GDP}} = 0.4$), we view the baseline forecasts as better capturing low-frequency variation. For another earnings-growth series, we obtained aggregate annual data on analyst-report-based terminal growth expectations (typically at a longer horizon than the IBES LTG measure) from [Décaire and Guenzel \(2025\)](#), and we thank Paul Décaire and Marius Guenzel for sharing their data. In line with the other $\Delta g_{t,d}^*$ proxies, terminal growth expectations have declined in the U.S. since the early 2000's (the start of their sample).

While [Greenwald, Lettau, and Ludvigson \(2025\)](#) differ from us in their focus on profit-share effects on prices, our results echo some of their findings. They find that “essentially all of the increase in equity values relative to output from the mid-1990s to the end of the sample” can be accounted for by assuming a fixed ratio of market equity to earnings (p. 1093), indicating that shocks to contemporaneous earnings relative to output drive the vast majority of equity price (but not equity yield) movements. This is also reflected in the analysis of [Atkeson, Heathcote, and Perri \(2025\)](#), who find that increases in the ratio of cash flows to value added have affected valuation much more than the ratio of price to cash flows. While we do not have direct evidence on the importance of profit-share changes outside the U.S., [Atkeson, Heathcote, and Perri \(2025\)](#) also estimate (see their Figure 7) that the increase in cash flows to value added appears fairly specific to the U.S. data.

Finally, as noted in the main text, we rerun our higher-frequency return regressions in [Table 2](#), but now with changes in the two $g_{t,d}^*$ proxies (profit growth and LTG) as explanatory variables in addition to the change in expected output growth. Results are presented in [Table B.5](#) below. The results are very consistent with those presented in [Table 2](#), with the coefficient on $(\widehat{\Delta\rho_t^*})$ very close to the original estimate of -19.1. The loading on ΔLTG is also significant and positive. Overall, our main results continue to apply even when considering changes in the profit share.

B.5 Details on Simulations for Figure 7

[Figure 7](#) plots the distribution of average realized stock returns across 100,000 simulations of an artificial economy. We now explain how each simulation is conducted. Each simulation has two periods (0,1), and one period amounts to 30 years. We simulate expected consumption growth $g_1 = g_0 + \varepsilon_1^g$ (abusing notation slightly), pure discounting terms $\rho_1 = \rho_0 + \varepsilon_1^\rho$, and SDF entropy $L_{1,M} = L_{0,M} + \varepsilon_1^L$, as well as dividends $d_1 = d_0 + \mu^d + \varepsilon_1^D$. The equity risk premium is $rp_t = \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - r_{t+1}^f = \theta_t + L_{t,M}$, where θ_0 is chosen to match the average equity premium and θ_1 is either equal to θ_0 or adjusts to ensure that the equity yield does not drop below 0.5%. Based on these variables, we calculate the ex ante interest rate as $r_t^f = \rho_t + \gamma g_t - L_{t,M}$, where we choose $\gamma = 2$ to match the empirically observed value. We calculate equity yields as $ey_t = r_t^f + rp_t - \lambda g_t$, and set the leverage parameter $\lambda = 2$ to match the empirical observation that earnings yields are close to orthogonal to movements in growth rates. At time zero, our calibration implies $rp_0 = 4\%$ (relative to long-term yields), $r_0^f = 4\%$, $g_0 = 2\%$, such that $ey_0 = 4\%$, and the duration of the stock market is 25 years (which we view as somewhat conservative).

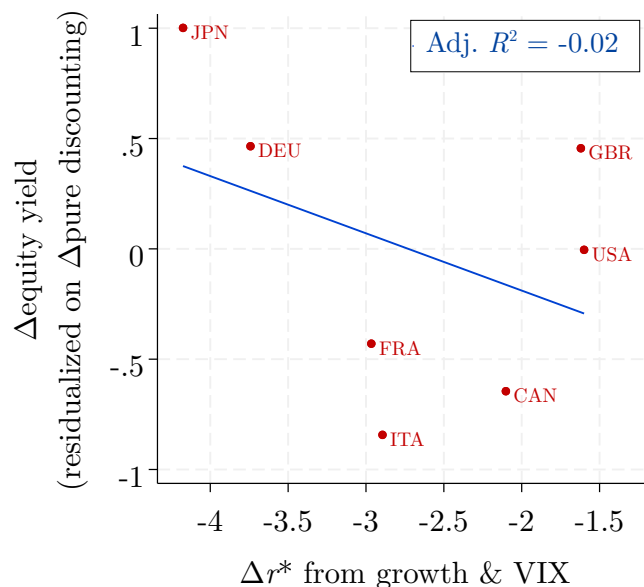
The shocks ε are all drawn from heavy-tailed t -distributions with 5 degrees of freedom. We choose the standard deviations of ε^ρ and ε^g to match the standard deviations in our

global panel, and we choose the standard deviation of ε^L such that the volatility of realized bond returns matches the annualized volatility of realized 30-year bond returns in the long U.S. sample; we estimate this empirical counterpart to be around 2% based on data from Robert Shiller’s website. Finally, we set the volatility of ε^D such that the volatility of realized stock returns matches the volatility of realized 30-year stock returns in the long U.S. sample, again based on data from Robert Shiller’s website. We conduct 100,000 simulations, and we calculate and plot the distribution of realized excess returns relative to all three benchmarks described in the main text.

B.6 Additional Results: Tables and Figures

See below for additional results referenced in the text, and see the main text for further discussion and explanation of each of the tables and figures below.

Figure B.1: Residualized Equity Yield Changes vs. Growth and Uncertainty



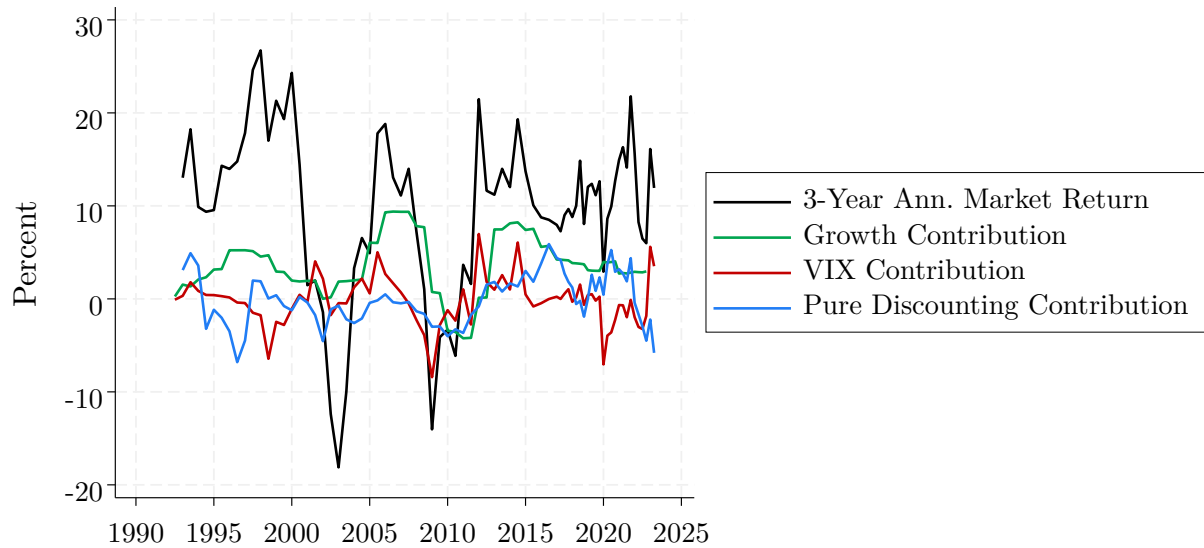
Notes: This figure plots the country-level change in equity yields against changes in interest rates from growth rates and uncertainty, where the equity yield change has now been residualized against the pure discounting change shown in the left panel of [Figure 3](#), $\Delta \hat{\rho}_{t,j}^*$. The sample is 1990–2023, or the longest available span for the given country. See [Figure 3](#) for additional details.

Table B.1: Regressions for Three-Year Changes in Trend Real Rates

	(1) U.S.	(2) All	(3) All
Change in expected growth $\Delta g_{t,j}^*$	0.5** (0.2)	0.3** (0.1)	0.3** (0.1)
Change in uncertainty $\Delta \text{VIX}_{t,j}^2$	-4.3** (2.1)	-0.9 (1.8)	β_j
Constant	-0.3*** (0.1)	-0.5*** (0.1)	-0.5*** (0.1)
Country FEs	X	✓	✓
Country-Specific $\text{VIX}_{t,j}^2$ Loading	✓	X	✓
Obs.	74	784	784
R^2	0.17	0.05	0.06
Within R^2	—	0.02	0.04

Notes: This table shows estimated OLS coefficients in the regression (21), along with standard errors in parentheses. In column (1), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (2)–(3), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. In column (3), the country-specific loadings on the squared VIX, β_j , are statistically significant at the 10% level for 6 of the 12 countries in our sample, and at the 5% level for 3 of the 12 countries (including the U.S.). The sample is 1990–2023, or the longest available span for the given country.

Figure B.2: Decomposition of U.S. Value-Weighted Equity Returns



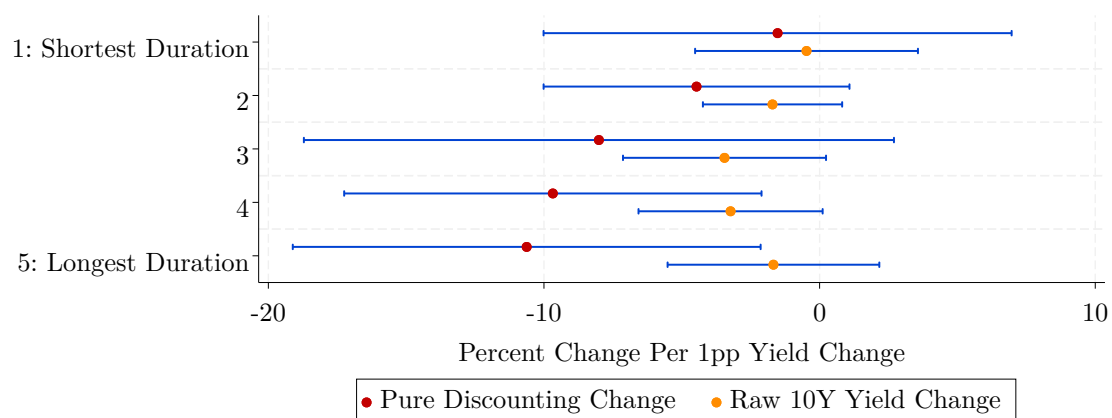
Notes: This figure shows three-year annualized average returns for the value-weighted U.S. stock market, along with estimated underlying contributors. The growth contribution is the estimated coefficient $\hat{\pi}_g$ in (22) times the three-year change $\Delta g_{t,j}^*$, plus the estimated contribution of contemporaneous cash flows. For this additional contribution, we estimate a regression identical to (22) but with realized three-year log dividend growth $\Delta d_{t,j}$ as an additional predictor; the contemporaneous cash-flow contribution is equal to $\Delta d_{t,j}$ times its estimated coefficient. Each of the other two contribution terms in the figure is equal to the corresponding predictor times its estimated coefficient in (22). The coefficient estimates are taken from column (2) of Table 2, but divided by 3 (e.g., the pure discount loading is -6.3 rather than -19.1). This is to account for the use of annualized returns in this plot, whereas the outcome variable for Table 2 is cumulative non-annualized returns. The pure discounting predictor for (22) is obtained from the first-stage estimation in (21).

Table B.2: Forecasting Regressions for Future Three-Year Market Returns

	(1)	(2)	(3)
10y yield	0.08 (0.38)		
Survey-based r_t^*		0.50 (0.68)	
Pure discounting term $\hat{\rho}_t^*$			2.08*** (0.61)
Country FEs	✓	✓	✓
Obs.	1,050	842	842
R^2	0.06	0.03	0.06
Within R^2	0.00	0.00	0.03

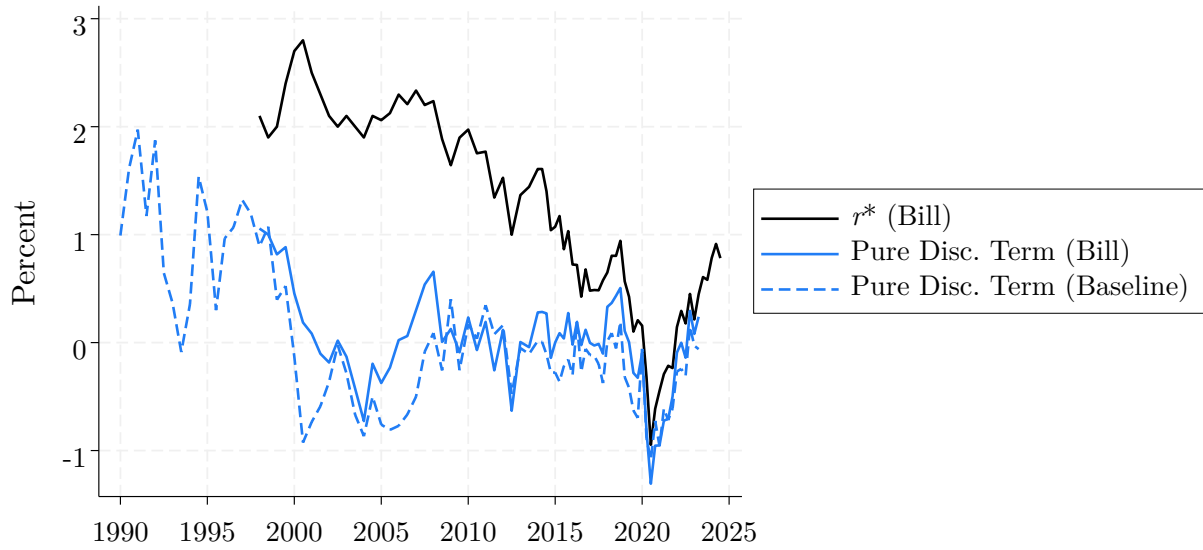
Notes: This table shows coefficient estimates from forecasting regressions $r_{t,t+3}^{\text{mkt}} = \alpha + \beta X_t + \varepsilon_{t,t+3}$, where $r_{t,t+3}^{\text{mkt}}$ is the country-level annualized three-year market return, and X_t is an ex ante predictor. The first column uses the 10-year nominal yield as the predictor variable, using data from each country’s central bank. The second column uses our survey-based measure of the trend real rate r_t^* as predictor. The third column uses our estimated pure discounting term $\hat{\rho}_t^*$, estimated using (19)–(20) following the main specification in column (3) of Table 1. Each regression includes country fixed effects, and all standard errors are clustered by country and date. The sample is 1990–2023, or the longest available span for the given country.

Figure B.3: Portfolio Exposures to Pure Discount Rate Changes: Global Stocks



Notes: This figure repeats the analysis shown in [Figure 5](#) using the full global sample of stocks. We form duration-sorted portfolios in the international panel following [Gormsen and Lazarus \(2023\)](#), and then we estimate the same regressions as in [Figure 5](#), with country-level fixed effects. The sample is 1990–2023.

Figure B.4: U.S. Estimation Results: Alternative with Short-Rate Forecasts



Notes: This figure replicates the U.S. decomposition in [Figure 2](#), but with r^* measured using the Consensus forecast of real bill yields (i.e., the forecasted nominal 3-month Treasury bill rate in year $t+5$, minus forecasted inflation at the same horizon). The black line shows this bill-based r^* series, and the solid blue line shows the implied pure discounting term, estimated as in [Figure 2](#) but with the bill-based r^* as the outcome variable in the decomposition regression. The dashed blue line shows the same pure discounting residual as plotted in [Figure 2](#). The short-rate sample is 1998–2023.

Table B.3: Robustness of r^* Decomposition: 10-Year Forecast Horizon

	(1)	(2)	(3)	(4)	(5)
	Baseline	SPF-Based	Short-Term r^*	GARCH Vol.	Unc. Index
<i>Inputs changed:</i>	—	r^*, g^*	r^*	L^*	L^*
<i>Robustness of Figure 3: Regressions of Δey^* on $\Delta \rho^*$ (G7)</i>					
Slope	1.01*** (0.21)	—	0.84*** (0.15)	0.92*** (0.19)	1.03*** (0.22)
Intercept	0.12 (0.30)	—	0.35 (0.25)	0.23 (0.30)	0.21 (0.30)
Adj. R^2	0.77	—	0.72	0.84	0.77
<i>Robustness of ρ^*: Time-Series Properties (U.S.)</i>					
$\Delta \rho^*$ over sample	-0.92	-1.35	-1.03	-0.94	-0.96
Corr($\rho^*, \rho_{\text{baseline}}^*$)	1.00	0.73	0.75	0.99	1.00

Notes: In this version of Table 3, all relevant Consensus forecast inputs are taken at the 10-year horizon (6-to-10-year average forecasts) rather than the main baseline five-year forward horizon. All alternative regression inputs are as described in Table 3. As in that table, all r^* decompositions follow the specification in column (3) of Table 1 for measures available across countries (columns (1), (3)–(5)), and they follow column (1) of Table 1 for the U.S.-only measure (col. (2)). Col. (1) uses the baseline specification, with the 10-year forecasts used for r^* and g^* . Col. (2) replaces the Consensus data with SPF forecasts for the average 10-year real yield and real GDP growth over the next decade. Col. (3) replaces the Consensus-based r^* with the cross-country short-term natural-rate estimates of Del Negro et al. (2019). These cross-country panel r^* estimates are annual and run through 2019; the U.S. change in ρ^* is extended to the end of the sample using a projection of this ρ^* onto the baseline ρ^* . Col. (4) replaces VIX^2 with a country-level 6-month GARCH(1,1) forecast of equity return variance. Col. (5) replaces VIX^2 with the country-level Baker, Bloom, and Davis (2016) uncertainty index. The top panel shows cross-country (G7) regressions of full-sample changes in equity yields on changes in ρ^* , for all available specifications. Standard errors in parentheses are obtained via block bootstrap by year with 10,000 draws. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The bottom panel shows U.S. time-series results. The first row shows the full-sample change in the estimated ρ^* . The second row shows the correlation between the alternative measure of ρ^* and this table’s version of the baseline measure from column (1). The sample is 1990–2023, or the longest available span for the given country.

Table B.4: Robustness of r^* Decomposition: 6-to-10-Year Growth Forecasts

	(1)	(2)	(3)	(4)	(5)
	Baseline	SPF-Based	Short-Term r^*	GARCH Vol.	Unc. Index
<i>Inputs changed:</i>	—	r^*, g^*	r^*	L^*	L^*
<i>Robustness of Figure 3: Regressions of Δey^* on $\Delta \rho^*$ (G7)</i>					
Slope	0.92*** (0.19)	—	0.84*** (0.15)	0.85*** (0.16)	0.94*** (0.19)
Intercept	-0.11 (0.28)	—	0.35 (0.25)	0.03 (0.27)	-0.02 (0.27)
Adj. R^2	0.83	—	0.72	0.87	0.83
<i>Robustness of ρ^*: Time-Series Properties (U.S.)</i>					
$\Delta \rho^*$ over sample	-0.93	-1.35	-0.96	-0.98	-0.97
Corr($\rho^*, \rho^*_{\text{baseline}}$)	1.00	0.75	0.78	0.99	0.99

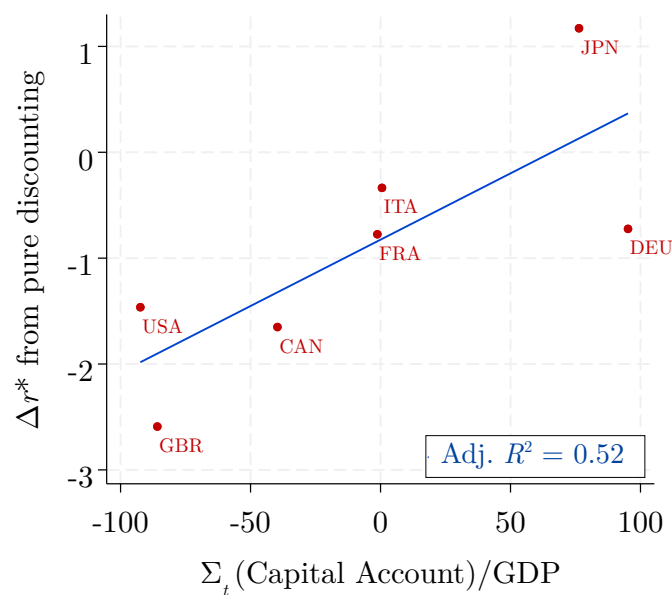
Notes: In this version of Table 3, expected growth g^* is measured using the Consensus 6-to-10-year average growth forecast rather than the main baseline five-year forecast. The trend real rate r^* is still measured at the five-year forward horizon, so this version effectively imposes that expected yields in five years depend on expected *forward-looking* growth rates at that point. All alternative regression inputs are as described in Table 3. As in that table, all r^* decompositions follow the specification in column (3) of Table 1 for measures available across countries (columns (1), (3)–(5)), and they follow column (1) of Table 1 for the U.S.-only measure (col. (2)). Col. (1) uses the baseline specification, with the 6-to-10-year forecast used for g^* . Col. (2) replaces the Consensus data with SPF forecasts for the average 10-year real yield and real GDP growth over the next decade. Col. (3) replaces the Consensus-based r^* with the cross-country short-term natural-rate estimates of Del Negro et al. (2019). These cross-country panel r^* estimates are annual and run through 2019; the U.S. change in ρ^* is extended to the end of the sample using a projection of this ρ^* onto the baseline ρ^* . Col. (4) replaces VIX^2 with a country-level 6-month GARCH(1,1) forecast of equity return variance. Col. (5) replaces VIX^2 with the country-level Baker, Bloom, and Davis (2016) uncertainty index. The top panel shows cross-country (G7) regressions of full-sample changes in equity yields on changes in ρ^* , for all available specifications. Standard errors in parentheses are obtained via block bootstrap by year with 10,000 draws. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The bottom panel shows U.S. time-series results. The first row shows the full-sample change in the estimated ρ^* . The second row shows the correlation between the alternative measure of ρ^* and this table’s version of the baseline measure from column (1). The sample is 1990–2023, or the longest available span for the given country.

Table B.5: Three-Year Return Regressions: Profit-Share Robustness

	(1)	(2)	(3)	(4)
	U.S.	U.S.	U.S.	U.S.
$\Delta 10\text{y yield}$	4.19 (3.51)			
$\Delta \text{pure discount } (\widehat{\Delta \rho_t^*})$		-19.1** (7.64)	-24.6*** (8.95)	-17.9*** (6.61)
$\Delta \text{exp. growth}$		-1.49 (14.0)	-15.3 (16.7)	-2.30 (12.7)
$\Delta \text{exp. profit growth}$			-0.14 (4.02)	
ΔLTG				5.94*** (2.03)
$\Delta \text{VIX}^2 \times 100$		-3.08** (1.33)	-4.62*** (1.46)	-2.88*** (1.09)
Obs.	74	74	58	74
R^2	0.04	0.20	0.47	0.36

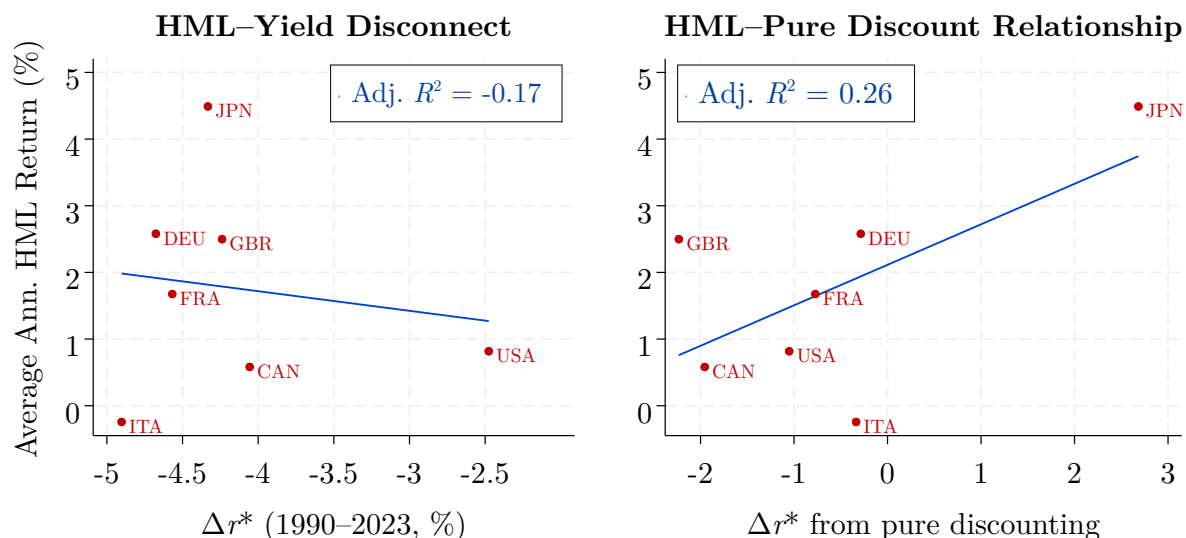
Notes: This table replicates the analysis in [Table 2](#) for U.S. data, but with additional predictor variables for other measures related to equity dividend growth. See the notes for [Table 2](#) for details on the estimation and inference, and see [Section 4.2](#) for descriptions of the additional predictor variables.

Figure B.5: Cross-Country Pure Discounting Changes vs. Total Capital Flows



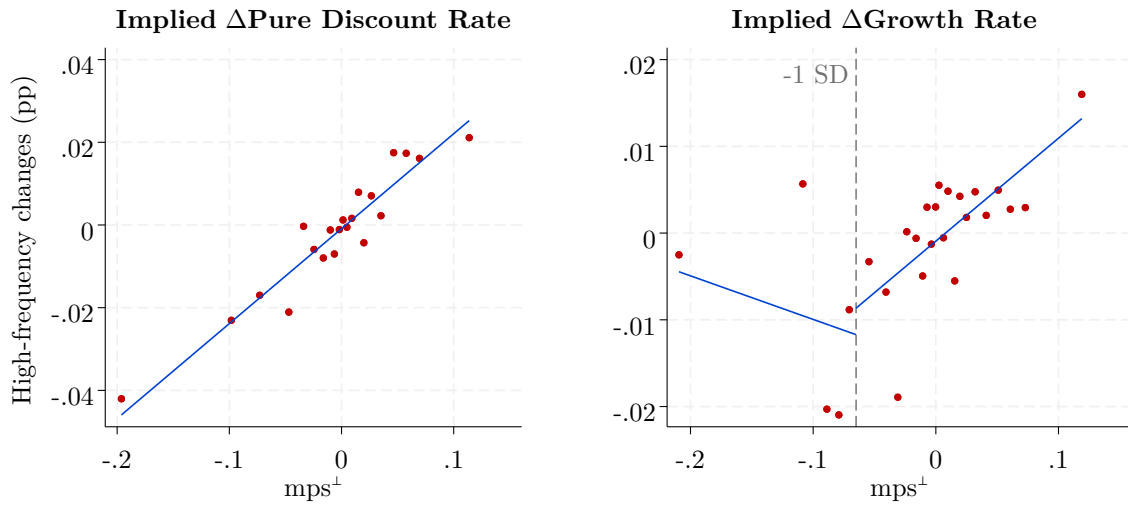
Notes: The vertical axis shows the country-level change in the pure discounting residual, as plotted on the horizontal axis in the first panel of Figure 3. The horizontal axis shows the cumulative sum of that country's net capital account balance as a share of annual GDP, measured from the IMF's "Net Financial Account" series. The sample is 1990–2023, or the longest available span for the given country.

Figure B.6: Interest Rates and Value Returns: Long-Term Global Evidence



Notes: The left panel plots the country-level average annual return on the high-minus-low (HML) book-to-market value factor versus the change in estimated trend real rate r^* over the 1990–2023 sample. Following Fama and French (1993), each country’s HML factor is constructed based on a 2×3 size and book-to-market double sort, with returns calculated as the average of the high-minus-low return for small and large firms. See Gormsen and Lazarus (2023) for details. The right panel plots the same average annual HML return against the change in the pure discounting term estimated using (19)–(20) following the main specification in column (3) of Table 1. The sample is 1990–2023. Note that HML returns are available starting in 1990 for all countries, whereas the equity yield samples start later than this in some cases (see Appendix B.1). As a result, the Δr^* terms in this figure may differ relative to Figure 3, given the earlier start date for the changes calculated in this figure.

Figure B.7: Changes in Decomposition Terms Around Monetary Policy Shocks



Notes: This figure shows binned scatter plots of the change in the pure discount rate $\widehat{\Delta\rho}_t^*$ (left panel) and the expected growth rate Δg_t (right panel) against the [Bauer and Swanson \(2023b\)](#) orthogonalized monetary policy shock $\text{mps}_t^\perp$ in percentage points (pp). The changes in pure discount rates and expected growth are recovered from (24)–(25) using the high-frequency (30-minute) stock-price and interest-rate changes. The sample is 1990–2023. In the left panel, the line of best fit is estimated from all available observations using the same regression as in [Table 4](#), column (2). In the right panel, we show two fit lines: one for large expansionary shocks for which $\text{mps}_t^\perp$ is less than one standard deviation below its sample mean ($\text{mps}_t^\perp < -0.065$), and one for the remaining shocks, separated by the vertical dashed line. Large expansionary shocks generate conventional policy responses, whereby larger easing shocks lead to higher expected growth (as in the left part of the panel). The remaining shocks otherwise generate a positive slope consistent with an information effect (as in the right part of the panel).